

# Collider signatures of the $SO(5) \times U(1)$ gauge-Higgs unification

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## Abstract

Collider signatures of the  $SO(5) \times U(1)$  gauge-Higgs unification model in the Randall-Sundrum warped space are explored. Gauge couplings of quarks and leptons receive small corrections from the fifth dimension whose effects are tested by the precision data. It is found that the forward-backward asymmetries in  $e^+e^-$  collisions on the  $Z$  pole are well explained in a wide range of the warp factor  $z_L$ , but the model is consistent with the branching fractions of  $Z$  decay only for large  $z_L \gtrsim 10^{15}$ . Kaluza-Klein (KK) spectra of gauge bosons, quarks, and leptons as well as gauge and Higgs couplings of low-lying KK excited states are determined. Right-handed quarks and leptons have larger couplings to the KK gauge bosons than left-handed ones. Production rates of Higgs bosons and KK states at Tevatron, LHC and ILC are evaluated. The first KK  $Z$  has a mass 1130 GeV with a width 422 GeV for  $z_L = 10^{15}$ . The current limit on the  $Z'$  production at Tevatron and LHC indicates  $z_L > 10^{15}$ . A large effect of parity violation appears in the difference between the rapidity distributions of  $e^+$  and  $e^-$  in the decay of the first KK  $Z$ . The first KK gauge bosons decay into light and heavy quarks evenly.

# 1 Introduction

One of the biggest issues in current physics is to find the Higgs boson and pin down its properties. The mechanism of electroweak (EW) symmetry breaking is yet to be uncovered. It is not clear if the EW symmetry is spontaneously broken in a way described in the standard model (SM). The search for the Higgs boson is carried on at Tevatron and LHC. The forthcoming result will tell us whether or not the SM scenario of the Higgs boson with a mass  $< 200$  GeV is correct.

Many alternative models have been proposed with new physics beyond the standard model. The most popular scenario in this category is supersymmetry hidden at the TeV scale. The Higgs boson is absent in the Higgsless model in which Kaluza-Klein (KK) modes of gauge bosons in the fifth dimension unitarize the theory,[1, 2] whereas the Higgs boson appears as a pseudo-Nambu-Goldstone boson in the little Higgs model.[3, 4, 5] In the gauge-Higgs unification scenario the Higgs boson is unified with 4D gauge fields, appearing as a part of the extra-dimensional component of gauge potentials.

In the present paper we focus on phenomenological implications and predictions of gauge-Higgs unification,[6]-[12] particularly of the  $SO(5) \times U(1)$  model in the Randall-Sundrum (RS) warped space.[13]-[18] The Higgs boson is nothing but a four-dimensional fluctuation mode of the Wilson line phase  $\theta_H$  representing an Aharonov-Bohm phase in the fifth dimension. It has been shown in a class of the  $SO(5) \times U(1)$  models that the energy density is minimized at  $\theta_H = \pm \frac{1}{2}\pi$  [17] where the Higgs boson becomes absolutely stable. There emerges the  $H$  parity invariance. Among low energy particles only the Higgs boson is  $H$  parity odd, whereas all other SM particles are  $H$  parity even.[19, 20]

One immediate consequence is that Higgs bosons become the dark matter of the universe. From the WMAP data the Higgs boson mass  $m_H$  is estimated around 70 GeV.[19] This value does not contradict with the LEP2 bound  $m_H > 114$  GeV, as the  $ZZH$  coupling exactly vanishes. In collider experiments Higgs bosons can be produced in pairs. However, they appear as missing energies and momenta as they do not decay.[21, 22]

How can we test the model at colliders? We examine this question by analyzing the precision data of gauge couplings of quarks and leptons, Higgs pair production at LHC and ILC, KK spectra of various fields, and production and decay of the first KK modes of gauge bosons at Tevatron and LHC. The model has one free parameter, the warp factor  $z_L$  of the RS space. It will be found that the present data from colliders prefer large  $z_L > 10^{15}$

whereas the Higgs mass  $\sim 70$  GeV accounting for the dark matter is obtained with  $z_L \sim 10^5$  in the current model. The production of the first KK mode of the  $Z$  boson with a mass around 1130 GeV and a width 422 GeV for  $z_L = 10^{15}$  at LHC will be one of the robust signals of the model.

The  $SO(5) \times U(1)$  gauge-Higgs unification model at  $\theta_H = \pm \frac{1}{2}\pi$  has similarity to the Higgsless model in such processes as  $WW$  scattering at the tree level as the Higgs boson contribution is absent due to the vanishing  $WWH$  coupling.[1] It has been also discussed that the Higgs boson in the model has correspondence to the holographic pseudo-Goldstone boson,[23, 24] resembling the little Higgs model. The stable Higgs boson serving as dark matter has similarity to a second Higgs boson in the inert Higgs doublet model with new parity.[25]-[28] We would like to stress that the current model can make many definitive, quantitative predictions by starting from a concrete action, to be compared with other predictions.[29]-[46]

The paper is organized as follows. The model is introduced in Section 2, and Kaluza-Klein (KK) expansions of various fields are summarized in Section 3. In Section 4 gauge couplings of quarks and leptons are determined, and are compared with the precision data for forward-backward asymmetries in  $e^+e^-$  annihilation on the  $Z$  resonance and partial decay widths of the  $Z$  boson. In Section 5 pair production of Higgs bosons at LHC and ILC is examined. The spectrum of KK towers of gauge bosons, quarks, and leptons are determined in Section 6. Couplings of quarks and leptons to KK gauge bosons are evaluated in Section 7. In Section 8 signals of the first KK  $Z$  boson at Tevatron and LHC are discussed. Section 9 is devoted to conclusions.

## 2 Model

The  $SO(5) \times U(1)$  gauge-Higgs unification scenario was proposed by Agashe, Contino, and Pomarol.[13] We analyze phenomenological consequences of the model given in refs. [18] and [20]. The model without leptons was introduced in ref. [17]. It has been shown that the model has  $H$  parity invariance which leads to the stable Higgs boson.[19, 20]

The model is defined in the Randall-Sundrum (RS) warped space with a metric

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (2.1)$$

where  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ ,  $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$ , and  $\sigma(y) = k|y|$  for  $|y| \leq L$ . The Planck and TeV branes are located at  $y = 0$  and  $y = L$ , respectively. The bulk region

$0 < y < L$  is anti-de Sitter (AdS) spacetime with a cosmological constant  $\Lambda = -6k^2$ . The warp factor  $z_L \equiv e^{kL} \gg 1$  is a parameter to be specified. The Kaluza-Klein (KK) mass scale is given by  $m_{\text{KK}} = \pi k/(z_L - 1) \sim \pi k z_L^{-1}$ , which turns out  $840 \sim 1470 \text{ GeV}$  for  $z_L = 10^5 \sim 10^{15}$ .

The model consists of  $SO(5) \times U(1)_X \times SU(3)_c$  gauge fields  $(A_M, B_M, G_M)$ , bulk fermions  $\Psi_a$ , brane fermions  $\hat{\chi}_{\alpha R}$ , and brane scalar  $\Phi$ . The bulk part of the action is given by

$$S_{\text{bulk}} = \int d^5x \sqrt{-G} \left[ -\text{tr} \frac{1}{4} F^{(A)MN} F_{MN}^{(A)} - \frac{1}{4} F^{(B)MN} F_{MN}^{(B)} - \text{tr} \frac{1}{2} F^{(C)MN} F_{MN}^{(C)} + \sum_a i \bar{\Psi}_a \mathcal{D}(c_a) \Psi_a \right],$$

$$\mathcal{D}(c_a) = \Gamma^A e_A^M \left( \partial_M + \frac{1}{8} \omega_{MBC} [\Gamma^B, \Gamma^C] - i g_A A_M - i g_B Q_{Xa} B_M - i g_C Q^{\text{color}} G_M \right) - c_a \sigma'(y). \quad (2.2)$$

The gauge fixing and ghost terms associated with the three gauge groups have been suppressed.  $F_{MN}^{(A)} = \partial_M A_N - \partial_N A_M - i g_A [A_M, A_N]$ ,  $F_{MN}^{(B)} = \partial_M B_N - \partial_N B_M$ , and  $F_{MN}^{(C)} = \partial_M G_N - \partial_N G_M - i g_C [G_M, G_N]$ .  $Q^{\text{color}} = 1$  or  $0$  for quark or lepton multiplets, respectively. The  $SO(5)$  gauge fields  $A_M$  are decomposed as  $A_M = \sum_{a_L=1}^3 A_M^{a_L} T^{a_L} + \sum_{a_R=1}^3 A_M^{a_R} T^{a_R} + \sum_{\hat{a}=1}^4 A_M^{\hat{a}} T^{\hat{a}}$ , where  $T^{a_L, a_R}$  ( $a_L, a_R = 1, 2, 3$ ) and  $T^{\hat{a}}$  ( $\hat{a} = 1, 2, 3, 4$ ) are the generators of  $SO(4) \simeq SU(2)_L \times SU(2)_R$  and  $SO(5)/SO(4)$ , respectively. In the fermion part  $\bar{\Psi} = i \Psi^\dagger \Gamma^0$  and Dirac  $\Gamma^M$  matrices are given by

$$\Gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad \sigma^\mu = (1, \vec{\sigma}), \quad \bar{\sigma}^\mu = (-1, \vec{\sigma}). \quad (2.3)$$

All of the bulk fermions belong to the vector **(5)** representation of  $SO(5)$ . The  $c_a$  term in Eq. (2.2) gives a bulk kink mass, where  $\sigma'(y) = k\epsilon(y)$  is a periodic step function with a magnitude  $k$ . The dimensionless parameter  $c_a$  plays an important role in controlling profiles of fermion wave functions.

The orbifold boundary conditions at  $y_0 = 0$  and  $y_1 = L$  are given by

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) P_j^{-1},$$

$$\begin{pmatrix} B_\mu \\ B_y \end{pmatrix} (x, y_j - y) = \begin{pmatrix} B_\mu \\ -B_y \end{pmatrix} (x, y_j + y),$$

$$\Psi_a(x, y_j - y) = P_j \Gamma^5 \Psi_a(x, y_j + y),$$

$$P_j = \text{diag}(-1, -1, -1, -1, +1) . \quad (2.4)$$

The  $SO(5) \times U(1)_X$  symmetry is reduced to  $SO(4) \times U(1)_X \simeq SU(2)_L \times SU(2)_R \times U(1)_X$  by the orbifold boundary conditions. It is known that various orbifold boundary conditions fall into a finite number of equivalence classes of boundary conditions.[8, 47, 48] The physical symmetry of the true vacuum in each equivalence class of boundary conditions is dynamically determined at the quantum level.

The 4D Higgs field, which is a doublet both in  $SU(2)_L$  and in  $SU(2)_R$ , appears as a zero mode in the  $SO(5)/SO(4)$  part of the fifth dimensional component of the vector potential  $A_y^{\hat{a}}(x, y)$ . Without loss of generality one assumes  $\langle A_y^{\hat{a}} \rangle \propto \delta^{a4}$  when the EW symmetry is spontaneously broken. The zero modes of  $A_y^{\hat{a}}$  ( $a = 1, 2, 3$ ) are absorbed by  $W$  and  $Z$  bosons. The Wilson line phase  $\theta_H$  is given by

$$\exp \left\{ \frac{i}{2} \theta_H \cdot 2\sqrt{2} T^{\hat{4}} \right\} = \exp \left\{ ig_A \int_0^L dy \langle A_y \rangle \right\} . \quad (2.5)$$

The 4D neutral Higgs field  $H(x)$  appears as [39]

$$A_y^{\hat{4}}(x, y) = \{ \theta_H f_H + H(x) \} u_H(y) + \dots ,$$

$$f_H = \frac{2}{g_A} \sqrt{\frac{k}{z_L^2 - 1}} = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}} . \quad (2.6)$$

Here the wave function of the 4D Higgs boson is given by  $u_H(y) = [2k/(z_L^2 - 1)]^{1/2} e^{2ky}$  for  $0 \leq y \leq L$  and  $u_H(-y) = u_H(y) = u_H(y + 2L)$ .  $g_w = g_A/\sqrt{L}$  is the dimensionless 4D  $SU(2)_L$  coupling.

For each generation two vector multiplets  $\Psi_1$  and  $\Psi_2$  for quarks and two vector multiplets  $\Psi_3$  and  $\Psi_4$  for leptons are introduced. Each vector multiplet,  $\Psi$ , is decomposed into one  $(\frac{1}{2}, \frac{1}{2})$ ,  $\check{\Psi}$ , and one  $(0, 0)$  of  $SU(2)_L \times SU(2)_R$ . We denote  $\Psi_a$ 's, for the third generation, as

$$\begin{aligned} \Psi_1 &= (\check{\Psi}_1, t')_{2/3} , \quad \check{\Psi}_1 = \begin{pmatrix} T & t \\ B & b \end{pmatrix} \equiv (Q_1, q) , \\ \Psi_2 &= (\check{\Psi}_2, b')_{-1/3} , \quad \check{\Psi}_2 = \begin{pmatrix} U & X \\ D & Y \end{pmatrix} \equiv (Q_2, Q_3) , \\ \Psi_3 &= (\check{\Psi}_3, \tau')_{-1} , \quad \check{\Psi}_3 = \begin{pmatrix} \nu_\tau & L_{1X} \\ \tau & L_{1Y} \end{pmatrix} \equiv (\ell, L_1) , \\ \Psi_4 &= (\check{\Psi}_4, \nu'_\tau)_0 , \quad \check{\Psi}_4 = \begin{pmatrix} L_{2X} & L_{3X} \\ L_{2Y} & L_{3Y} \end{pmatrix} \equiv (L_2, L_3) . \end{aligned} \quad (2.7)$$

Subscripts 2/3 etc. represent  $U(1)_X$  charges,  $Q_X$ , of  $\Psi_a$ 's.  $q$ ,  $Q_j$ ,  $\ell$ , and  $L_j$  are  $SU(2)_L$  doublets. The electromagnetic charge  $Q_{\text{EM}}$  is given by

$$Q_{\text{EM}} = T^{3L} + T^{3R} + Q_X . \quad (2.8)$$

Each  $\Psi_a$  has its bulk mass parameter  $c_a$ . Consistent results are obtained by taking  $c_1 = c_2 \equiv c_q$  and  $c_3 = c_4 \equiv c_\ell$  for each generation.

The additional brane fields are introduced on the Planck brane at  $y = 0$ . The brane scalar field  $\Phi$  belongs to  $(0, \frac{1}{2})$  of  $SU(2)_L \times SU(2)_R$  with  $Q_X = -\frac{1}{2}$ , whereas the right-handed brane fermions  $\hat{\chi}_{\alpha R}^q$  and  $\hat{\chi}_{\alpha R}^\ell$  belong to  $(\frac{1}{2}, 0)$ . The brane fermions are

$$\begin{aligned} \hat{\chi}_{1R}^q &= \begin{pmatrix} \hat{T}_R \\ \hat{B}_R \end{pmatrix}_{7/6}, \quad \hat{\chi}_{2R}^q = \begin{pmatrix} \hat{U}_R \\ \hat{D}_R \end{pmatrix}_{1/6}, \quad \hat{\chi}_{3R}^q = \begin{pmatrix} \hat{X}_R \\ \hat{Y}_R \end{pmatrix}_{-5/6}, \\ \hat{\chi}_{1R}^\ell &= \begin{pmatrix} \hat{L}_{1XR} \\ \hat{L}_{1YR} \end{pmatrix}_{-3/2}, \quad \hat{\chi}_{2R}^\ell = \begin{pmatrix} \hat{L}_{2XR} \\ \hat{L}_{2YR} \end{pmatrix}_{1/2}, \quad \hat{\chi}_{3R}^\ell = \begin{pmatrix} \hat{L}_{3XR} \\ \hat{L}_{3YR} \end{pmatrix}_{-1/2}. \end{aligned} \quad (2.9)$$

Subscripts 7/6 etc. represent  $Q_X$  charges of  $\hat{\chi}_R$ 's. The brane part of the action is given by

$$\begin{aligned} S_{\text{brane}} &= \int d^5x \sqrt{-G} \delta(y) \left\{ - (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda_\Phi (\Phi^\dagger \Phi - w^2)^2 \right. \\ &\quad + \sum_{\alpha=1}^3 \left( \hat{\chi}_{\alpha R}^{q\dagger} i \bar{\sigma}^\mu D_\mu \hat{\chi}_{\alpha R}^q + \hat{\chi}_{\alpha R}^{\ell\dagger} i \bar{\sigma}^\mu D_\mu \hat{\chi}_{\alpha R}^\ell \right) \\ &\quad - i \left[ \kappa_1^q \hat{\chi}_{1R}^{q\dagger} \tilde{\Psi}_{1L} \tilde{\Phi} + \tilde{\kappa}^q \hat{\chi}_{2R}^{q\dagger} \tilde{\Psi}_{1L} \Phi + \kappa_2^q \hat{\chi}_{2R}^{q\dagger} \tilde{\Psi}_{2L} \tilde{\Phi} + \kappa_3^q \hat{\chi}_{3R}^{q\dagger} \tilde{\Psi}_{2L} \Phi - (\text{h.c.}) \right] \\ &\quad \left. - i \left[ \tilde{\kappa}^\ell \hat{\chi}_{3R}^{\ell\dagger} \tilde{\Psi}_{3L} \tilde{\Phi} + \kappa_1^\ell \hat{\chi}_{1R}^{\ell\dagger} \tilde{\Psi}_{3L} \Phi + \kappa_2^\ell \hat{\chi}_{2R}^{\ell\dagger} \tilde{\Psi}_{4L} \tilde{\Phi} + \kappa_3^\ell \hat{\chi}_{3R}^{\ell\dagger} \tilde{\Psi}_{4L} \Phi - (\text{h.c.}) \right] \right\}, \\ D_\mu \Phi &= \left( \partial_\mu - i g_A \sum_{a_R=1}^3 A_\mu^{a_R} T^{a_R} + i \frac{1}{2} g_B B_\mu \right) \Phi, \quad \tilde{\Phi} = i \sigma_2 \Phi^*, \\ D_\mu \hat{\chi} &= \left( \partial_\mu - i g_A \sum_{a_L=1}^3 A_\mu^{a_L} T^{a_L} - i Q_X g_B B_\mu - i g_C Q^{\text{color}} G_\mu \right) \hat{\chi}. \end{aligned} \quad (2.10)$$

The action  $S_{\text{brane}}$  is manifestly invariant under  $SU(2)_L \times SU(2)_R \times U(1)_X$ . The Yukawa couplings above exhaust all possible ones preserving the symmetry.

The non-vanishing vev  $\langle \Phi^t \rangle = (0, w)$  has two important consequences. It is assumed only that  $w \gg m_{\text{KK}}$ . Firstly the  $SU(2)_R \times U(1)_X$  symmetry is spontaneously broken down to  $U(1)_Y$  and the zero modes of four-dimensional gauge fields of  $SU(2)_R \times U(1)_X$  become massive except for the  $U(1)_Y$  part. They acquire masses of  $O(m_{\text{KK}})$  as a result of the

effective change of boundary conditions for low-lying modes in the Kaluza-Klein towers. Secondly the non-vanishing vev  $w$  induces mass couplings between brane fermions and bulk fermions;

$$\begin{aligned}
S_{\text{brane}}^{\text{mass}} = \int d^5x \sqrt{-G} \delta(y) & \left\{ - \sum_{\alpha=1}^3 i\mu_{\alpha}^q (\hat{\chi}_{\alpha R}^{q\dagger} Q_{\alpha L} - Q_{\alpha L}^{\dagger} \hat{\chi}_{\alpha R}^q) - i\tilde{\mu}^q (\hat{\chi}_{2R}^{q\dagger} q_L - q_L^{\dagger} \hat{\chi}_{2R}^q) \right. \\
& \left. - \sum_{\alpha=1}^3 i\mu_{\alpha}^{\ell} (\hat{\chi}_{\alpha R}^{\ell\dagger} L_{\alpha L} - L_{\alpha L}^{\dagger} \hat{\chi}_{\alpha R}^{\ell}) - i\tilde{\mu}^{\ell} (\hat{\chi}_{3R}^{\ell\dagger} \ell_L - \ell_L^{\dagger} \hat{\chi}_{3R}^{\ell}) \right\} , \\
\frac{\mu_{\alpha}^q}{\kappa_{\alpha}^q} = \frac{\tilde{\mu}^q}{\tilde{\kappa}^q} = \frac{\mu_{\alpha}^{\ell}}{\kappa_{\alpha}^{\ell}} = \frac{\tilde{\mu}^{\ell}}{\tilde{\kappa}^{\ell}} = w , & \tag{2.11}
\end{aligned}$$

Assuming that all  $\mu^2 \gg m_{\text{KK}}$ , all of the exotic zero modes of the bulk fermions acquire large masses of  $O(m_{\text{KK}})$ . It has been shown that all of the 4D anomalies associated with  $SU(2)_L \times SU(2)_R \times U(1)_X$  gauge symmetry are cancelled.[18] The  $SU(2)_L \times U(1)_Y$  is further broken down to  $U(1)_{\text{EM}}$  by the Hosotani mechanism. The spectrum of the resultant light particles are the same as in the standard model. The generation mixing can be explained by considering matrix couplings of  $\kappa_j$  and  $\tilde{\kappa}$ , particularly of  $\kappa_2^q, \tilde{\kappa}^q, \kappa_3^{\ell}, \tilde{\kappa}^{\ell}$ .

The parameters of the model relevant for low energy physics are  $k$ ,  $z_L = e^{kL}$ ,  $g_A$ ,  $g_B$ , the bulk mass parameters  $(c_q, c_{\ell})$  and the brane mass ratios  $(\tilde{\mu}^q/\mu_2^q, \tilde{\mu}^{\ell}/\mu_3^{\ell})$ . All other parameters are irrelevant at low energies, provided that  $w$ ,  $\mu^2$ 's are much larger than  $m_{\text{KK}}$ . The value of  $\theta_H$  is determined dynamically to be  $\pm \frac{1}{2}\pi$  where the EW symmetry is spontaneously broken.[20] Three of the four parameters  $k$ ,  $z_L = e^{kL}$ ,  $g_A$ ,  $g_B$  are determined from the  $Z$  boson mass  $m_Z$ , the weak gauge coupling  $g_w$ , and the Weinberg angle  $\sin^2 \theta_W$ . The one parameter, say,  $z_L$  remains free.

When the generation mixing is turned off in the fermion sector, the bulk mass  $c_q$  and the ratio  $\tilde{\mu}^q/\mu_2^q$  in each generation are determined from the two quark masses, and  $c_{\ell}$  and  $\tilde{\mu}^{\ell}/\mu_3^{\ell}$  from the two lepton masses. As  $m_{\nu_e} \ll m_e$ , all of the results discussed below do not depend on the unknown value of  $m_{\nu_e}$  very much. The generation mixing can be incorporated by considering 3-by-3 matrices for the brane couplings  $\kappa$ 's, or equivalently for the brane masses  $\mu$ 's.

Once the value of  $z_L$  is specified, all the relevant parameters of the model are determined. The spectra of particles and their KK towers, their wave functions in the fifth dimension, and all interaction couplings can be predicted. The mass of the 4D Higgs boson,  $m_H$ , is determined from the effective potential  $V_{\text{eff}}(\theta_H)$ . It was found that  $m_H$  is about

$70 \sim 135 \text{ GeV}$  for  $z_L = 10^5 \sim 10^{15}$ . [20] Conversely the remaining one parameter  $z_L$  is fixed, once the Higgs boson mass  $m_H$  is given.

As typical reference values we take the warp factors  $z_L = 10^5, 10^{10}, 10^{15}$ . The values in Table 1 are taken as input parameters. The masses of quarks and charged leptons except for  $t$  quark are quoted from Ref. [51]. The masses of  $Z$  boson and  $t$  quark are the central values in the Particle Data Group review [52]. The couplings  $\alpha$  and  $\alpha_s$  are also quoted from Ref. [52]. In the present analysis, the neutrino masses have negligible effects.

The parameter  $\sin^2 \theta_W$  is determined from  $\chi^2$  fit of forward-backward asymmetries in  $e^+e^-$  annihilation and branching ratios in the  $Z$  decay as explained below. We find the best fit with  $\sin^2 \theta_W = 0.2284, 0.2303, 0.2309$  for  $z_L = 10^5, 10^{10}, 10^{15}$ , respectively. Since complete one-loop analysis is not available in the gauge-Higgs unification scenario at the moment, there remains ambiguity in the value of  $\sin^2 \theta_W$ .

Table 1: Input parameters for the masses and couplings of the model. The masses are in an unit of GeV. All masses except for  $m_t$  are at the  $m_Z$  scale.

$m_Z$	91.1876	$\alpha_s(m_Z)$	0.1176
$m_u$	$1.27 \times 10^{-3}$	$m_{\nu_e}$	$1 \times 10^{-12}$
$m_c$	0.619	$m_{\nu_\mu}$	$9 \times 10^{-12}$
$m_t$	171.17	$m_{\nu_\tau}$	$5.0309 \times 10^{-11}$
$m_d$	$2.90 \times 10^{-3}$	$m_e$	$0.486570161 \times 10^{-3}$
$m_s$	0.055	$m_\mu$	$102.7181359 \times 10^{-3}$
$m_b$	2.89	$m_\tau$	1.74624

### 3 Kaluza-Klein expansion

With the orbifold boundary condition (2.4) the effective potential  $V_{\text{eff}}(\theta_H)$  is minimized at  $\theta_H = \frac{1}{2}\pi$ . To develop perturbation theory around  $\theta_H = \frac{1}{2}\pi$ , it is most convenient to move to the twisted gauge  $\tilde{A}_M = \Omega A_M \Omega^{-1} + (i/g_A)\Omega \partial_M \Omega^{-1}$  in which  $\langle \tilde{A}_y \rangle = 0$ , or  $\tilde{\theta}_H = 0$ . We choose  $\Omega$  preserving the boundary condition at the TeV brane;

$$\Omega(y) = \exp \left\{ \frac{i\pi g_A f_H}{2} \int_y^L dy' u_H(y') \cdot T^{\hat{4}} \right\}. \quad (3.1)$$

In the twisted gauge the orbifold boundary condition  $\{P_0, P_1\}$  is changed to  $\{\tilde{P}_0, \tilde{P}_1\}$  where

$$\tilde{P}_0 = \Omega(-y)P_0\Omega(y)^{-1} = \text{diag}(-1, -1, -1, +1, -1) \neq P_0,$$

$$\tilde{P}_1 = \Omega(L - y)P_1\Omega(L + y)^{-1} = \text{diag}(-1, -1, -1, -1, +1) = P_1 . \quad (3.2)$$

The two sets  $\{P_0, P_1\}$  and  $\{\tilde{P}_0, \tilde{P}_1\}$  are in the same equivalence class of boundary conditions.[8, 47, 48, 49] Although the boundary conditions are different, physics remains the same as a result of dynamics of the Wilson line phase. Note that  $\Omega(L) = 1$ , but

$$\Omega(0) = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 0 & 1 \\ & & & -1 & 0 \end{pmatrix} \quad (3.3)$$

so that the brane interactions take more complicated form than in the original gauge.

In the previous paper it was shown that the model has  $H$  parity ( $P_H$ ) invariance, and  $H$  parity is assigned to all 4D fields.[20]  $P_H$  interchanges  $SU(2)_L$  and  $SU(2)_R$  and flips the sign of  $T^{\hat{4}}$  in the bulk.  $P_H$  transformation is generated by  $T^\alpha \rightarrow \Omega_H T^\alpha \Omega_H^{-1}$  where  $\Omega_H = \text{diag}(1, 1, 1, -1, 1)$  in the twisted gauge. The  $P_H$  symmetry is similar to the  $P_{LR}$  symmetry discussed by Agashe, Contino, Da Rold and Pomarol [32], which protects the  $T$  parameter and  $Zb\bar{b}$  coupling from radiative corrections. The neutral Higgs boson is the lightest particle of odd  $P_H$  so that it becomes stable.

In the twisted gauge the four-dimensional components of gauge fields are expanded as

$$\begin{aligned} \tilde{A}_\mu(x, z) &= \hat{W}_\mu + \hat{W}_\mu^\dagger + \hat{Z}_\mu + \hat{A}_\mu^\gamma + \hat{W}'_\mu + \hat{W}'_\mu{}^\dagger + \hat{Z}'_\mu + \hat{A}_\mu^{\hat{4}} , \\ \hat{W}_\mu &= \sum_n W_\mu^{(n)} \left\{ h_{W^{(n)}}^L T^{-L} + h_{W^{(n)}}^R T^{-R} + h_{W^{(n)}}^\wedge T^{\hat{4}} \right\} , \\ \hat{Z}_\mu &= \sum_n Z_\mu^{(n)} \left\{ h_{Z^{(n)}}^L T^{3L} + h_{Z^{(n)}}^R T^{3R} + h_{Z^{(n)}}^\wedge T^{\hat{3}} \right\} , \\ \hat{A}_\mu^\gamma &= \sum_n A_\mu^{\gamma(n)} \left\{ h_{\gamma^{(n)}}^L T^{3L} + h_{\gamma^{(n)}}^R T^{3R} \right\} , \\ \hat{W}'_\mu &= \sum_n W_\mu'^{(n)} \left\{ h_{W'^{(n)}}^L T^{-L} + h_{W'^{(n)}}^R T^{-R} \right\} , \\ \hat{Z}'_\mu &= \sum_n Z_\mu'^{(n)} \left\{ h_{Z'^{(n)}}^L T^{3L} + h_{Z'^{(n)}}^R T^{3R} \right\} , \\ \hat{A}_\mu^{\hat{4}} &= \sum_n A_\mu^{\hat{4}(n)} h_{A^{(n)}} T^{\hat{4}} , \\ \tilde{B}_\mu(x, z) &= \sum_n Z_\mu^{(n)} h_{Z^{(n)}}^B + \sum_n A_\mu^{\gamma(n)} h_{\gamma^{(n)}}^B . \end{aligned} \quad (3.4)$$

Here  $T^\pm = (T^1 \pm iT^2)/\sqrt{2}$ . The  $W$  and  $Z$  bosons and the photon  $\gamma$  correspond to  $W_\mu^{(0)}$ ,  $Z_\mu^{(0)}$  and  $A_\mu^{\gamma(0)}$ , respectively. Unless confusion arises, we will omit the superscript (0) for representing the lowest mode. The mixing angle between  $SO(5)$  and  $U(1)_X$  is related to the Weinberg angle by  $\sin^2 \theta_W \equiv s_\phi^2/(1+s_\phi^2)$  where  $s_\phi = g_B/\sqrt{g_A^2 + g_B^2}$ . All mode functions  $h(z)$  are tabulated in Appendix A. They are expressed in terms of Bessel functions

$$\begin{aligned} C(z; \lambda) &= \frac{\pi}{2} \lambda z z_L F_{1,0}(\lambda z, \lambda z_L) , & C'(z; \lambda) &= \frac{\pi}{2} \lambda^2 z z_L F_{0,0}(\lambda z, \lambda z_L) , \\ S(z; \lambda) &= -\frac{\pi}{2} \lambda z F_{1,1}(\lambda z, \lambda z_L) , & S'(z; \lambda) &= -\frac{\pi}{2} \lambda^2 z F_{0,1}(\lambda z, \lambda z_L) , \\ F_{\alpha,\beta}(u, v) &= J_\alpha(u) Y_\beta(v) - Y_\alpha(u) J_\beta(v) . \end{aligned} \quad (3.5)$$

For the photon ( $\lambda_0 = 0$ ),  $h_{\gamma(0)}^L = h_{\gamma(0)}^R$  is constant.

The mass spectrum  $m_n = k\lambda_n$  of each KK tower is determined by the corresponding eigenvalue equations:

$$\begin{aligned} W_\mu^{(n)} &: 2S(1; \lambda_n) C'(1; \lambda_n) + \lambda_n = 0 , \\ Z_\mu^{(n)} &: 2S(1; \lambda_n) C'(1; \lambda_n) + \lambda_n(1 + s_\phi^2) = 0 , \\ W_\mu'^{(n)}, Z_\mu'^{(n)} &: C(1; \lambda_n) = 0 , \\ A_\mu^{\gamma(n)} &: C'(1; \lambda_n) = 0 , \\ A_\mu^{\hat{1}(n)} &: S(1; \lambda_n) = 0 . \end{aligned} \quad (3.6)$$

The Weinberg angle  $\theta_W$  is determined by global fit of various quantities. In the present paper  $\sin^2 \theta_W$  is determined from  $\chi^2$  fit of forward-backward asymmetries in  $e^+e^-$  annihilation and  $Z$  boson decay. With  $m_Z$  and  $z_L$  as an input, the AdS curvature  $k$  and the  $W$  boson mass at the tree level,  $m_W^{\text{tree}}$ , are determined.

Similarly the fifth-dimensional components  $A_z$  and  $B_z$  are expanded as

$$\begin{aligned} \tilde{A}_z(x, z) &= \sum_{a=1}^3 \sum_{n=1}^{\infty} S^{a(n)} h_S^{LR}(\lambda_n) \frac{T^{a_L} + T^{a_R}}{\sqrt{2}} + \sum_{n=0}^{\infty} H^{(n)} h_H^\wedge(\lambda_n) T^{\hat{4}} \\ &+ \sum_{a=1}^3 \sum_{n=1}^{\infty} D_-^{a(n)} h_D^{LR}(\lambda_n) \frac{T^{a_L} - T^{a_R}}{\sqrt{2}} + \sum_{a=1}^3 \sum_{n=1}^{\infty} \hat{D}^{a(n)} h_D^\wedge(\lambda_n) T^{\hat{a}} , \\ \tilde{B}_z(x, z) &= \sum_{n=1}^{\infty} B^{(n)} h_B(\lambda_n) . \end{aligned} \quad (3.7)$$

$H(x) = H^{(0)}(x)$  is the 4D neutral Higgs boson. The mass spectrum of each KK tower is given by

$$S^{a(n)}, B^{(n)} : C'(1; \lambda_n) = 0 ,$$

$$\begin{aligned}
D_-^{a(n)} &: C(1; \lambda_n) = 0, \\
\hat{D}^{a(n)} &: S'(1; \lambda_n) = 0, \\
H^{(n)} &: S(1; \lambda_n) = 0,
\end{aligned} \tag{3.8}$$

For the bulk fields  $H$  parity is assigned from the behavior under the transformation  $\{T^{a_L}, T^{a_R}, T^{\hat{a}}, T^{\hat{4}}\} \rightarrow \{T^{a_R}, T^{a_L}, T^{\hat{a}}, -T^{\hat{4}}\}$ . It interchanges  $SU(2)_L$  and  $SU(2)_R$  and flips the direction of  $T^{\hat{4}}$ . Accordingly  $P_H$  odd fields are

$$P_H \text{ odd} : W'_\mu{}^{(n)}, Z'_\mu{}^{(n)}, A_\mu^{\hat{4}(n)}, H^{(n)}, D_-^{a(n)}. \tag{3.9}$$

Other fields are  $P_H$  even.

As for the fermions, a consistent model is obtained with the bulk mass parameters  $c_1 = c_2 \equiv c_q$  and  $c_3 = c_4 \equiv c_\ell$ . Let us first consider the multiplets containing quarks, namely,  $\Psi_1$  and  $\Psi_2$  in (2.7) and  $\hat{\chi}_{1R}^q, \hat{\chi}_{2R}^q, \hat{\chi}_{3R}^q$ , in (2.9). They are classified in terms of electric charge  $Q_E = \frac{5}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{4}{3}$ .

We recall that components of  $\check{\Psi}$  in (2.7) are related to the components  $\Psi^k$  ( $k = 1 \sim 5$ ) in the vectorial representation by

$$\check{\Psi} = \begin{pmatrix} \check{\Psi}_{11} & \check{\Psi}_{12} \\ \check{\Psi}_{21} & \check{\Psi}_{22} \end{pmatrix} = -\frac{1}{\sqrt{2}} \begin{pmatrix} \Psi^2 + i\Psi^1 & -\Psi^4 - i\Psi^3 \\ \Psi^4 - i\Psi^3 & \Psi^2 - i\Psi^1 \end{pmatrix}. \tag{3.10}$$

$\Psi^4$  and  $\Psi^5$  couple with  $A_z^{\hat{4}}$  or  $\theta_H$ . Conversely we have, for the third generation in the twisted gauge,

$$\begin{pmatrix} \tilde{\Psi}_1^1 \\ \tilde{\Psi}_1^2 \\ \tilde{\Psi}_1^3 \\ \tilde{\Psi}_1^4 \\ \tilde{\Psi}_1^5 \end{pmatrix} = \begin{pmatrix} i(\tilde{T} - \tilde{b})/\sqrt{2} \\ -(\tilde{T} + \tilde{b})/\sqrt{2} \\ -i(\tilde{B} + \tilde{t})/\sqrt{2} \\ -(\tilde{B} - \tilde{t})/\sqrt{2} \\ t' \end{pmatrix}, \quad \begin{pmatrix} \tilde{\Psi}_2^1 \\ \tilde{\Psi}_2^2 \\ \tilde{\Psi}_2^3 \\ \tilde{\Psi}_2^4 \\ \tilde{\Psi}_2^5 \end{pmatrix} = \begin{pmatrix} i(\tilde{U} - \tilde{Y})/\sqrt{2} \\ -(\tilde{U} + \tilde{Y})/\sqrt{2} \\ -i(\tilde{D} + \tilde{X})/\sqrt{2} \\ -(\tilde{D} - \tilde{X})/\sqrt{2} \\ b' \end{pmatrix}, \tag{3.11}$$

$\Omega_H$  transformation gives  $(\tilde{\Psi}^1, \tilde{\Psi}^2, \tilde{\Psi}^3, \tilde{\Psi}^4, \tilde{\Psi}^5) \rightarrow (\tilde{\Psi}^1, \tilde{\Psi}^2, \tilde{\Psi}^3, -\tilde{\Psi}^4, \tilde{\Psi}^5)$ . The  $\tilde{\Psi}^4$  component is  $P_H$  odd, whereas other components are  $P_H$  even.

The  $Q_E = \frac{5}{3}$  sector consists of  $T$  in  $\Psi_1$  and  $\hat{T}_R$  in  $\hat{\chi}_{1R}^q$ . The  $Q_E = -\frac{4}{3}$  sector consists of  $Y$  in  $\Psi_2$  and  $\hat{Y}_R$  in  $\hat{\chi}_{3R}^q$ . There are no light modes in these two sectors.

The  $Q_E = \frac{2}{3}$  sector consists of  $B, t, t'$  in  $\Psi_1$ ,  $U$  in  $\Psi_2$ ,  $\hat{B}_R$  in  $\hat{\chi}_{1R}^q$  and  $\hat{U}_R$  in  $\hat{\chi}_{2R}^q$ . The bulk fermions have the following Kaluza-Klein expansion.

$$\begin{pmatrix} \tilde{U}_L \\ (\tilde{B}_L + \tilde{t}_L)/\sqrt{2} \\ \tilde{t}'_L \end{pmatrix} (x, z) = \sqrt{k} \sum_{n=0}^{\infty} \begin{pmatrix} a_U^{(n)} C_L(z; \lambda_n, c_t) \\ a_{B+t}^{(n)} C_L(z; \lambda_n, c_t) \\ a_{t'}^{(n)} S_L(z; \lambda_n, c_t) \end{pmatrix} \psi_{\frac{2}{3}(+),L}^{(n)}(x),$$

$$\begin{aligned}
\begin{pmatrix} \tilde{U}_R \\ (\tilde{B}_R + \tilde{t}_R)/\sqrt{2} \\ \tilde{t}'_R \end{pmatrix} (x, z) &= \sqrt{k} \sum_{n=0}^{\infty} \begin{pmatrix} a_U^{(n)} S_R(z; \lambda_n, c_t) \\ a_{B+t}^{(n)} S_R(z; \lambda_n, c_t) \\ a_{t'}^{(n)} C_R(z; \lambda_n, c_t) \end{pmatrix} \psi_{\frac{2}{3}(+),R}^{(n)}(x) , \\
\begin{pmatrix} (\tilde{B}_L - \tilde{t}_L)/\sqrt{2} \\ (\tilde{B}_R - \tilde{t}_R)/\sqrt{2} \end{pmatrix} (x, z) &= \sqrt{k} \sum_{n=1}^{\infty} a_{B-t}^{(n)} \begin{pmatrix} C_L(z; \lambda_n, c_t) t_{(-),L}^{(n)}(x) \\ S_R(z; \lambda_n, c_t) t_{(-),R}^{(n)}(x) \end{pmatrix} .
\end{aligned} \tag{3.12}$$

Here  $c_t$  is the bulk kink mass for the third generation  $(\Psi_1, \Psi_2)$ , and

$$\begin{aligned}
\begin{pmatrix} C_L \\ S_L \end{pmatrix} (z; \lambda, c) &= \pm \frac{\pi}{2} \lambda \sqrt{z z_L} F_{c+\frac{1}{2}, c \mp \frac{1}{2}}(\lambda z, \lambda z_L) , \\
\begin{pmatrix} C_R \\ S_R \end{pmatrix} (z; \lambda, c) &= \mp \frac{\pi}{2} \lambda \sqrt{z z_L} F_{c-\frac{1}{2}, c \pm \frac{1}{2}}(\lambda z, \lambda z_L) .
\end{aligned} \tag{3.13}$$

$\psi_{\frac{2}{3}(+)}^{(n)}(x)$  fields are  $P_H$  even, while  $t_{(-)}^{(n)}(x)$  fields are  $P_H$  odd.  $\{\psi_{\frac{2}{3}(+)}^{(n)}(x)\}$  contains three KK towers, including the KK tower  $t_{(+)}^{(n)}(x)$  of the top quark. The brane fields  $\hat{B}_R$  and  $\hat{U}_R$  can be expressed in terms of the bulk fields.

The spectrum  $\lambda_n$  and mode coefficients  $a^{(n)}$  of the  $P_H$ -even towers satisfy

$$\det \hat{K} = 0 \quad , \quad \hat{K} \begin{pmatrix} a_U^{(n)} \\ \frac{1}{2} a_{B+t}^{(n)} \\ \frac{1}{\sqrt{2}} a_{t'}^{(n)} \end{pmatrix} = 0 , \tag{3.14}$$

where

$$\begin{aligned}
\hat{K} &= \begin{pmatrix} \lambda_n S_R - \frac{\mu_2^2}{2k} C_L & -\frac{\mu_2 \tilde{\mu}}{2k} C_L & \frac{\mu_2 \tilde{\mu}}{2k} S_L \\ 0 & \lambda_n S_R - \frac{\mu_1^2}{2k} C_L & (\lambda_n C_R - \frac{\mu_1^2}{2k} S_L) \\ -\frac{\mu_2 \tilde{\mu}}{2k} C_L & \lambda_n S_R - \frac{\tilde{\mu}^2}{2k} C_L & -(\lambda_n C_R - \frac{\tilde{\mu}^2}{2k} S_L) \end{pmatrix} , \\
C_{L,R} &= C_{L,R}(1; \lambda_n, c_t) \quad , \quad S_{L,R} = S_{L,R}(1; \lambda_n, c_t) .
\end{aligned} \tag{3.15}$$

Here we have suppressed a superscript  $q$  in  $\mu_j^q$ . There is one light mode (the top quark) with  $m_t = k\lambda_{t,0} \ll m_{\text{KK}}$ . When  $\mu_j^2, \tilde{\mu}^2 \gg m_{\text{KK}}$ , the spectrum of the top quark tower satisfies

$$2 \left( 1 + \frac{\tilde{\mu}^2}{\mu_2^2} \right) S_R(1; \lambda_{t,n}, c_t) S_L(1; \lambda_{t,n}, c_t) + 1 = 0 \tag{3.16}$$

for  $k\lambda_{t,n} \ll m_{\text{KK}}$ . A similar relation is obtained for the bottom quark  $m_b = k\lambda_{b,0}$ ;

$$2 \left( 1 + \frac{\mu_2^2}{\tilde{\mu}^2} \right) S_R(1; \lambda_{b,n}, c_t) S_L(1; \lambda_{b,n}, c_t) + 1 = 0 \tag{3.17}$$

for  $k\lambda_{b,n} \ll m_{\text{KK}}$ . With  $(m_t, m_b)$  given, Eqs. (3.16) and (3.17) determine the bulk mass  $c_t$  and the ratio  $\tilde{\mu}^2/\mu_2^2$ . We note that  $\tilde{\mu}^2/\mu_2^2 \sim m_b/m_t$  for  $m_b \ll m_t$ . The spectrum of the KK tower  $t_{(-)}^{(n)}(x)$  is determined by  $C_L(1; \lambda_n, c_t) = 0$ .

Parallel arguments apply to the  $Q_E = -\frac{1}{3}$  sector, which consists of  $b$  in  $\Psi_1$ ,  $D, X, b'$  in  $\Psi_2$ ,  $\hat{D}_R$  in  $\hat{\chi}_{2R}^q$  and  $\hat{X}_R$  in  $\hat{\chi}_{3R}^q$ . The bulk fields are expanded as

$$\begin{aligned} \begin{pmatrix} \tilde{b}_L \\ (\tilde{D}_L + \tilde{X}_L)/\sqrt{2} \\ \tilde{b}'_L \end{pmatrix} (x, z) &= \sqrt{k} \sum_{n=0}^{\infty} \begin{pmatrix} a_b^{(n)} C_L(z; \lambda_n, c_t) \\ a_{D+X}^{(n)} C_L(z; \lambda_n, c_t) \\ a_{b'}^{(n)} S_L(z; \lambda_n, c_t) \end{pmatrix} \psi_{-\frac{1}{3}(+),L}^{(n)}(x) , \\ \begin{pmatrix} \tilde{b}_R \\ (\tilde{D}_R + \tilde{X}_R)/\sqrt{2} \\ \tilde{b}'_R \end{pmatrix} (x, z) &= \sqrt{k} \sum_{n=0}^{\infty} \begin{pmatrix} a_b^{(n)} S_R(z; \lambda_n, c_t) \\ a_{D+X}^{(n)} S_R(z; \lambda_n, c_t) \\ a_{b'}^{(n)} C_R(z; \lambda_n, c_t) \end{pmatrix} \psi_{-\frac{1}{3}(+),R}^{(n)}(x) , \\ \begin{pmatrix} (\tilde{D}_L - \tilde{X}_L)/\sqrt{2} \\ (\tilde{D}_R - \tilde{X}_R)/\sqrt{2} \end{pmatrix} (x, z) &= \sqrt{k} \sum_{n=1}^{\infty} a_{D-X}^{(n)} \begin{pmatrix} C_L(z; \lambda_n, c_t) b_{(-),L}^{(n)}(x) \\ S_R(z; \lambda_n, c_t) b_{(-),R}^{(n)}(x) \end{pmatrix} . \end{aligned} \quad (3.18)$$

The equations and relations in the  $Q_E = -\frac{1}{3}$  sector are obtained from those in the  $Q_E = \frac{2}{3}$  sector by replacing  $(U, B, t, t')$  and  $(\mu_1, \mu_2, \tilde{\mu})$  by  $(b, D, X, b')$  and  $(\mu_3, \tilde{\mu}, \mu_2)$ , respectively.

Similar relations are obtained in the lepton sector. The generation mixing is incorporated by considering  $\mu_2, \tilde{\mu}$  in matrices.

## 4 4D gauge couplings

The 4D gauge couplings are obtained by performing overlapping integrals of wave functions. Generalizing the argument in Ref. [18], one can write, for the  $t$  and  $b$  quarks and the  $\tau$  and  $\nu_\tau$  leptons in the third generation, the couplings of the photon,  $W$  boson,  $Z$  boson, and gluon towers as

$$\begin{aligned} &\sum_n A_\mu^{\gamma(n)} \left\{ \frac{2}{3} (g_{tL}^{\gamma(n)} \bar{t}_L \gamma^\mu t_L + g_{tR}^{\gamma(n)} \bar{t}_R \gamma^\mu t_R) - \frac{1}{3} (g_{bL}^{\gamma(n)} \bar{b}_L \gamma^\mu b_L + g_{bR}^{\gamma(n)} \bar{b}_R \gamma^\mu b_R) \right. \\ &\quad \left. - (g_{\tau L}^{\gamma(n)} \bar{\tau}_L \gamma^\mu \tau_L + g_{\tau R}^{\gamma(n)} \bar{\tau}_R \gamma^\mu \tau_R) \right\} \\ &+ \sum_n \frac{1}{\sqrt{2}} W_\mu^{(n)} \left\{ g_{tb,L}^{W(n)} \bar{b}_L \gamma^\mu t_L + g_{tb,R}^{W(n)} \bar{b}_R \gamma^\mu t_R + g_{\tau,L}^{W(n)} \bar{\tau}_L \gamma^\mu \nu_{\tau L} + g_{\tau,R}^{W(n)} \bar{\tau}_R \gamma^\mu \nu_{\tau R} \right\} + \text{h.c.} \\ &+ \sum_n \frac{1}{\cos \theta_W} Z_\mu^{(n)} \left\{ g_{tL}^{Z(n)} \bar{t}_L \gamma^\mu t_L + g_{tR}^{Z(n)} \bar{t}_R \gamma^\mu t_R + g_{bL}^{Z(n)} \bar{b}_L \gamma^\mu b_L + g_{bR}^{Z(n)} \bar{b}_R \gamma^\mu b_R \right. \\ &\quad \left. + g_{\nu_\tau L}^{Z(n)} \bar{\nu}_{\tau L} \gamma^\mu \nu_{\tau L} + g_{\nu_\tau R}^{Z(n)} \bar{\nu}_{\tau R} \gamma^\mu \nu_{\tau R} + g_{\tau L}^{Z(n)} \bar{\tau}_L \gamma^\mu \tau_L + g_{\tau R}^{Z(n)} \bar{\tau}_R \gamma^\mu \tau_R \right\} \end{aligned}$$

$$\begin{aligned}
& + \sum_n G_\mu^{(n)a} \left\{ (g_{tL}^{G(n)} \bar{t}_L \gamma^\mu \frac{1}{2} \lambda^a t_L + g_{tR}^{G(n)} \bar{t}_R \gamma^\mu \frac{1}{2} \lambda^a t_R) \right. \\
& \quad \left. + (g_{bL}^{G(n)} \bar{b}_L \gamma^\mu \frac{1}{2} \lambda^a b_L + g_{bR}^{G(n)} \bar{b}_R \gamma^\mu \frac{1}{2} \lambda^a b_R) \right\}. \tag{4.1}
\end{aligned}$$

From the  $H$  parity invariance the  $W'$ ,  $Z'$  and  $A^4$  gauge boson towers do not couple to the quarks and leptons.

The couplings of the photon tower with the  $t$  and  $b$  quarks and  $\tau$  lepton are given, with  $h_{\gamma(n)}^L = h_{\gamma(n)}^R = (g_B/g_A)h_{\gamma(n)}^B \equiv h_{\gamma(n)}(z)$ , by

$$\begin{aligned}
g_{tL}^{\gamma(n)} &= g_A \int_1^{z_L} dz h_{\gamma(n)} \left\{ (a_U^2 + a_{B+t}^2) C_L(\lambda_t)^2 + a_{t'}^2 S_L(\lambda_t)^2 \right\}, \\
g_{bL}^{\gamma(n)} &= g_A \int_1^{z_L} dz h_{\gamma(n)} \left\{ (a_b^2 + a_{D+X}^2) C_L(\lambda_b)^2 + a_{b'}^2 S_L(\lambda_b)^2 \right\}, \\
g_{\tau L}^{\gamma(n)} &= g_A \int_1^{z_L} dz h_{\gamma(n)} \left\{ (a_{L_{3Y}}^2 + a_{\tau+L_{1X}}^2) C_L(\lambda_\tau)^2 + a_{\tau'}^2 S_L(\lambda_\tau)^2 \right\}. \tag{4.2}
\end{aligned}$$

Here  $C_L(\lambda_t) = C_L(z; \lambda_t, c_t)$  etc.. The formulas for right-handed fermions are obtained from those for the corresponding left-handed fermions by replacing  $C_L$  and  $S_L$  by  $S_R$  and  $C_R$ , respectively. The couplings of the  $W$  boson towers are given by

$$\begin{aligned}
g_{tb,L}^{W(n)} &= g_A \int_1^{z_L} dz \left\{ 2h_{W(n)} (a_b a_{B+t} + a_U a_{D+X}) C_L(\lambda_t) C_L(\lambda_b) \right. \\
& \quad \left. + \sqrt{2} h_{W(n)}^\wedge (a_B a_{t'} C_L(\lambda_b) S_L(\lambda_t) - a_{b'} a_U S_L(\lambda_b) C_L(\lambda_t)) \right\}, \\
g_{\tau,L}^{W(n)} &= g_A \int_1^{z_L} dz \left\{ 2h_{W(n)} (a_{\nu_\tau} a_{\tau+L_{1X}} + a_{L_{3Y}} a_{L_{2Y}+L_{3X}}) C_L(\lambda_\tau) C_L(\lambda_{\nu_\tau}) \right. \\
& \quad \left. + \sqrt{2} h_{W(n)}^\wedge (a_{L_{3Y}} a_{\nu_\tau'} C_L(\lambda_\tau) S_L(\lambda_{\nu_\tau}) - a_{\tau'} a_{\nu_\tau} S_L(\lambda_{\nu_\tau}) C_L(\lambda_\tau)) \right\} \tag{4.3}
\end{aligned}$$

where  $h_{W(n)} \equiv h_{W(n)}^L = h_{W(n)}^R$ . The couplings of the  $Z$  boson towers are parametrized as

$$\begin{aligned}
g_{tL,R}^{Z(n)} &= +\frac{1}{2} g_{tL,R}^{Z(n),T} - \frac{2}{3} g_{tL,R}^{Z(n),Q} \sin^2 \theta_W, \\
g_{bL,R}^{Z(n)} &= -\frac{1}{2} g_{bL,R}^{Z(n),T} + \frac{1}{3} g_{bL,R}^{Z(n),Q} \sin^2 \theta_W, \\
g_{\nu_\tau L,R}^{Z(n)} &= +\frac{1}{2} g_{\nu_\tau L,R}^{Z(n),T}, \\
g_{\tau L,R}^{Z(n)} &= -\frac{1}{2} g_{\tau L,R}^{Z(n),T} + g_{\tau L,R}^{Z(n),Q} \sin^2 \theta_W. \tag{4.4}
\end{aligned}$$

In the SM  $g_{fL}^{Z,T} = g_{fL}^{Z,Q} = g_{fR}^{Z,Q} = g_w$  and  $g_{fR}^{Z,T} = 0$ . In the current model, with the aid of (A.5), one finds that

$$g_{tL}^{Z(n),T} = \frac{\sqrt{2} g_A}{\sqrt{r_{Z(n)}}} \int_1^{z_L} dz \left\{ a_U^2 C_{Z(n)} C_L(\lambda_t)^2 - 2a_{B+t} a_{t'} \hat{S}_{Z(n)} C_L(\lambda_t) S_L(\lambda_t) \right\},$$

$$\begin{aligned}
g_{bL}^{Z^{(n)},T} &= \frac{\sqrt{2}g_A}{\sqrt{r_{Z^{(n)}}}} \int_1^{z_L} dz \left\{ a_b^2 C_{Z^{(n)}} C_L(\lambda_b)^2 + 2a_{D+X} a_{b'} \hat{S}_{Z^{(n)}} C_L(\lambda_b) S_L(\lambda_b) \right\} , \\
g_{\nu_\tau L}^{Z^{(n)},T} &= \frac{\sqrt{2}g_A}{\sqrt{r_{Z^{(n)}}}} \int_1^{z_L} dz \left\{ a_{\nu_\tau}^2 C_{Z^{(n)}} C_L(\lambda_{\nu_\tau})^2 - 2a_{L_{2Y}+L_{3X}} a_{\nu'_\tau} \hat{S}_{Z^{(n)}} C_L(\lambda_{\nu_\tau}) S_L(\lambda_{\nu_\tau}) \right\} , \\
g_{\tau L}^{Z^{(n)},T} &= \frac{\sqrt{2}g_A}{\sqrt{r_{Z^{(n)}}}} \int_1^{z_L} dz \left\{ a_{L_{3Y}}^2 C_{Z^{(n)}} C_L(\lambda_\tau)^2 + 2a_{\tau+L_{1X}} a_{\tau'} \hat{S}_{Z^{(n)}} C_L(\lambda_\tau) S_L(\lambda_\tau) \right\} , \\
g_{tL}^{Z^{(n)},Q} &= \frac{\sqrt{2}g_A}{\sqrt{r_{Z^{(n)}}}} \int_1^{z_L} dz C_{Z^{(n)}} \left\{ (a_U^2 + a_{B+t}^2) C_L(\lambda_t)^2 + a_{t'}^2 S_L(\lambda_t)^2 \right\} , \\
g_{bL}^{Z^{(n)},Q} &= \frac{\sqrt{2}g_A}{\sqrt{r_{Z^{(n)}}}} \int_1^{z_L} dz C_{Z^{(n)}} \left\{ (a_b^2 + a_{D+X}^2) C_L(\lambda_b)^2 + a_{b'}^2 S_L(\lambda_b)^2 \right\} , \\
g_{\tau L}^{Z^{(n)},Q} &= \frac{\sqrt{2}g_A}{\sqrt{r_{Z^{(n)}}}} \int_1^{z_L} dz C_{Z^{(n)}} \left\{ (a_{L_{3Y}}^2 + a_{\tau+L_{1X}}^2) C_L(\lambda_\tau)^2 + a_{\tau'}^2 S_L(\lambda_\tau)^2 \right\} , \tag{4.5}
\end{aligned}$$

where  $C_{Z^{(n)}} = C(z; \lambda_{Z^{(n)}})$  etc..

The couplings of the gluon tower  $g_{tI}^{G^{(n)}}$  and  $g_{bI}^{G^{(n)}}$  are obtained from the photon tower couplings  $g_{tI}^{\gamma^{(n)}}$  and  $g_{bI}^{\gamma^{(n)}}$  with the replacement of the five-dimensional coupling,  $g_{tI}^{G^{(n)}} = (g_C/g_A)g_{tI}^{\gamma^{(n)}}$  and  $g_{bI}^{G^{(n)}} = (g_C/g_A)g_{bI}^{\gamma^{(n)}}$ . The photon and gluon couplings are universal, that is,  $e = g_{tI}^{\gamma^{(0)}} = g_{bI}^{\gamma^{(0)}} = (g_A/\sqrt{L}) \sin \theta_W$ . The other couplings exhibit violation of the universality as evaluated below.

## 4.1 Zero mode couplings

The numerical values for the various gauge couplings are obtained with the input parameters given in Sec. 2. The couplings of  $W$  boson with quarks and leptons are tabulated in Tables 2. The ratios of the couplings to the 4D  $SU(2)$  coupling,  $g_f^{(W)}\sqrt{L}/g_A$ , have been tabulated. Except for  $tb$ , the couplings are almost universal. For the  $t_L b_L$  coupling the deviation amounts to  $2 \sim 6\%$  for  $z_L = 10^{15} \sim 10^5$ . The  $t_R b_R$  coupling is about  $0.09 \sim 0.3\%$  of the left-handed coupling for  $z_L = 10^{15} \sim 10^5$ .

The couplings of  $Z$  boson with quarks are tabulated in Tables 3. For reference, the tree-level values in the standard model,  $1/2 - (2/3)\sin^2 \theta_W$  and  $-1/2 + (1/3)\sin^2 \theta_W$  for left-handed quarks and  $-(2/3)\sin^2 \theta_W$  and  $(1/3)\sin^2 \theta_W$  for right-handed quarks are also listed. As we shall see below, the small violation of the universality gives a better fit to the forward-backward asymmetry data. As a general character for left-handed and right-handed quarks, it is found that the coupling of right-handed quarks for a small warp factor tends to deviate from the standard model values.

Table 2: The couplings of  $W$  boson with quarks and leptons,  $g_f^{(W)}\sqrt{L}/g_A$ .

$z_L$	$u_L d_L$	$c_L s_L$	$t_L b_L$	$\nu_{eL} e_L$	$\nu_{\mu L} \mu_L$	$\nu_{\tau L} \tau_L$
$10^{15}$	1.0053	1.0053	0.9816	1.0053	1.0053	1.0053
$10^{10}$	1.0079	1.0079	0.9730	1.0079	1.0079	1.0079
$10^5$	1.0154	1.0154	0.9470	1.0154	1.0154	1.0153

$z_L$	$u_R d_R$	$c_R s_R$	$t_R b_R$	$\nu_{eR} e_R$	$\nu_{\mu R} \mu_R$	$\nu_{\tau R} \tau_R$
$10^{15}$	$-5 \times 10^{-12}$	$-5 \times 10^{-8}$	-0.0009	$-3 \times 10^{-22}$	$-4 \times 10^{-16}$	$-6 \times 10^{-17}$
$10^{10}$	$-6 \times 10^{-12}$	$-7 \times 10^{-8}$	-0.0014	$-4 \times 10^{-22}$	$-7 \times 10^{-16}$	$-2 \times 10^{-13}$
$10^5$	$-9 \times 10^{-12}$	$-1 \times 10^{-7}$	-0.0031	$-5 \times 10^{-22}$	$-1 \times 10^{-18}$	$-2 \times 10^{-16}$

Table 3: The couplings of  $Z$  boson with quarks,  $g_f^{(Z)}\sqrt{L}/g_A$ .

$z_L$	$u_L$	$c_L$	$t_L$	$d_L$	$s_L$	$b_L$
$10^{15}$	0.3485	0.3485	0.3219	-0.4260	-0.4260	-0.4265
$10^{10}$	0.3501	0.3501	0.3086	-0.4276	-0.4276	-0.4288
$10^5$	0.3548	0.3548	0.2558	-0.4325	-0.4325	-0.4369
SM		0.3459			-0.4229	

$z_L$	$u_R$	$c_R$	$t_R$	$d_R$	$s_R$	$b_R$
$10^{15}$	-0.1562	-0.1562	-0.1835	0.07811	0.07809	0.07806
$10^{10}$	-0.1570	-0.1570	-0.2002	0.07852	0.07847	0.07839
$10^5$	-0.1595	-0.1593	-0.2656	0.07976	0.07965	0.07928
SM		-0.1541			0.07707	

The couplings of  $Z$  bosons with leptons are tabulated in Table 4. They are not very sensitive to the generation. As general tendency, the couplings deviate more from those in the standard model as the warp factor becomes smaller.

In the standard model the couplings of  $Z$  boson with fermions are described by the weak coupling and their quantum number, namely by  $(g_w/\cos\theta_W)(T^3 - Q\sin^2\theta_W)$ , at the tree level. In the present model they have an analogous form given by  $(1/\cos\theta_W)(g_{T,L}T^3 - g_{Q,L}Q\sin^2\theta_W)$  for left-handed fermions and by  $(1/\cos\theta_W)(g_{T,R} - g_{Q,R}Q\sin^2\theta_W)$  for right-handed fermions. Here  $g_T$  and  $g_Q$  depends on the flavor of fermions. It is found that  $g_{T,L} \approx g_{Q,L}$ . For right-handed fermions the absolute value of  $g_{T,R}\sqrt{L}/g_A$  are small for  $t$ -quark ( $\lesssim 10^{-2}$ ) and very small for the others ( $\lesssim 10^{-6}$ ), but  $g_{Q,R}\sqrt{L}/g_A$  can be of order  $\mathcal{O}(1)$  which leads to deviation from the standard model. The couplings of  $Z$  boson with right-handed neutrinos are very small as neutral fields have only the  $g_T$  component. For a similar reason the couplings of KK  $Z$  boson with right-handed neutrinos turn out very

Table 4: The couplings of  $Z$  bosons with leptons,  $g_f^{(Z)}\sqrt{L}/g_A$ .

$z_L$	$e_L$	$\mu_L$	$\tau_L$	$e_R$	$\mu_R$	$\tau_R$
$10^{15}$	-0.2710	-0.2710	-0.2710	0.2344	0.2343	0.2343
$10^{10}$	-0.2725	-0.2725	-0.2725	0.2356	0.2355	0.2354
$10^5$	-0.2771	-0.2771	-0.2771	0.2394	0.2391	0.2389
SM	-0.2688			0.2312		

$z_L$	$\nu_{eL}$	$\nu_{\mu L}$	$\nu_{\tau L}$	$\nu_{eR}$	$\nu_{\mu R}$	$\nu_{\tau R}$
$10^{15}$	0.5035	0.5035	0.5035	$-1.4 \times 10^{-13}$	$-7.2 \times 10^{-9}$	$-2.3 \times 10^{-6}$
$10^{10}$	0.5052	0.5052	0.5052	$-1.8 \times 10^{-13}$	$-9.7 \times 10^{-9}$	$-3.2 \times 10^{-6}$
$10^5$	0.5102	0.5102	0.5101	$-2.5 \times 10^{-13}$	$-1.5 \times 10^{-8}$	$-5.4 \times 10^{-6}$
SM	0.5			0		

small.

## 4.2 Forward-backward asymmetry

The forward-backward asymmetry on the  $Z$  resonance is given by

$$A_{FB}^f = \frac{3}{4} \left[ \frac{(g_{eL}^Z)^2 - (g_{eR}^Z)^2}{(g_{eL}^Z)^2 + (g_{eR}^Z)^2} \right] \left[ \frac{(g_{fL}^Z)^2 - (g_{fR}^Z)^2}{(g_{fL}^Z)^2 + (g_{fR}^Z)^2} \right], \quad (4.6)$$

which is evaluated from the gauge couplings given in the preceding subsection.  $A_{FB}^f$  does not depend on the absolute common magnitude of  $g_A$ , but sensitively depends on  $\sin^2 \theta_W$ . The branching fractions of various decay modes of the  $Z$  boson also sensitively depend on  $\sin^2 \theta_W$ . We have determined the value of  $\sin^2 \theta_W$  to minimize  $\chi^2$  of those experimental data as tabulated in Table 5. The value of  $\sin^2 \theta_W$  turns out a bit smaller than that in the standard model.

With given  $\sin^2 \theta_W$  the numerical values of  $A_{FB}^f$  are shown in Table 6.\* The experimental values are quoted from Ref. [52]. The current model gives rather good fit for the forward-backward asymmetries  $A_{FB}^f$ , though the fit to the  $Z$  decay fractions becomes poor for smaller values of  $z_L$ .

## 4.3 Decay width

The partial decay width of  $Z$  boson is given by

$$\Gamma(Z \rightarrow f\bar{f}) = \frac{m_Z}{12\pi \cos^2 \theta_W} F(g_{fL}^{(Z)}, g_{fR}^{(Z)}, m_f, m_Z),$$

---

\*The result for  $z_L = 10^{15}$  has been given in Ref. [43]. A slight difference in the numerical values is due to the different choice of the values of the input parameters.

Table 5:  $\chi^2$  fit for  $A_{FB}$  and  $Z$  decay fractions. The values of  $m_{KK}$ ,  $m_H$  and  $m_W^{\text{tree}}$  ( $W$  mass at the tree level) are also listed.

	# of data	$z_L = 10^{15}$	$10^{10}$	$10^5$	SM
$\sin^2 \theta_W$		0.2309	0.2303	0.2284	0.2312
$\chi^2 [A_{FB}]$	6	6.3	6.4	7.1	10.8
$\chi^2 [Z \text{ decay fractions}]$	8	16.5	37.7	184.5	13.6
Sum of two $\chi^2$	14	22.8	44.1	191.6	24.5
$m_{KK}$ (GeV)		1466	1193	836	
$m_H$ (GeV)		135	108	72	
$m_W^{\text{tree}}$ (GeV)		79.84	79.80	79.71	79.95

Table 6: The forward-backward asymmetry on the  $Z$  resonance,  $A_{FB}^f$ .

	Exp.	$z_L = 10^{15}$	$z_L = 10^{10}$	$z_L = 10^5$	SM
$e$	$0.0145 \pm 0.0025$	0.0156	0.0157	0.0159	$0.01633 \pm 0.00021$
$\mu$	$0.0169 \pm 0.0013$	0.0156	0.0157	0.0160	
$\tau$	$0.0188 \pm 0.0017$	0.0156	0.0158	0.0161	
$s$	$0.0976 \pm 0.0114$	0.1011	0.1014	0.1019	$0.1035 \pm 0.0007$
$c$	$0.0707 \pm 0.0035$	0.0720	0.0721	0.0725	$0.0739 \pm 0.0005$
$b$	$0.0992 \pm 0.0016$	0.1011	0.1014	0.1021	$0.1034 \pm 0.0007$

$$F(g_{fL}, g_{fR}, m_f, m_V) = \left\{ \frac{(g_{fL})^2 + (g_{fR})^2}{2} + 2g_{fL}g_{fR} \frac{m_f^2}{m_V^2} \right\} \sqrt{1 - \frac{4m_f^2}{m_V^2}}. \quad (4.7)$$

Here the couplings  $g_{fL}^{(Z)}$  and  $g_{fR}^{(Z)}$  are given in Tables 3 and 4. For quarks the formula should be multiplied by a factor  $3(1 + \alpha_s/\pi)$ .

For  $z_L = 10^{15}, 10^{10}, 10^5$ , the branching fractions in  $Z$  decay are shown in Table 7. The experimental values are quoted from Ref. [52]. The tree level prediction for branching fractions reproduces the pattern of the experimental values well for  $z_L = 10^{15}$ .

The total decay width  $\Gamma_{\text{tot}}$  depends on  $\alpha(m_Z)$ . The value of  $\alpha(m_Z)$  determined to fit the experimental value  $\Gamma_{\text{tot}}$  does not agree well with the value determined by renormalization group from the low energy data. For  $z_L = 10^{15}$ , for instance, one finds  $\alpha^{-1}(m_Z) = 130.5$ . At the moment one cannot reliably evaluate one loop corrections to  $\Gamma_{\text{tot}}$  in the gauge-Higgs unification scenario and this mismatch is understood within that error.

Table 7: The branching fractions in the  $Z$  boson decay. The invisible decay in the model means the decay into  $\nu_e + \nu_\mu + \nu_\tau$ .

$z_L$	$10^{15}$	$10^{10}$	$10^5$	Exp.
$e$ (%)	3.374	3.382	3.403	$3.363 \pm 0.004$
$\mu$ (%)	3.373	3.380	3.400	$3.366 \pm 0.007$
$\tau$ (%)	3.368	3.374	3.392	$3.370 \pm 0.008$
invisible (%)	19.99	19.95	19.82	$20.00 \pm 0.06$
$(u + c)/2$ (%)	11.93	11.94	11.95	$11.6 \pm 0.6$
$(d + s + b)/3$ (%)	15.34	15.34	15.36	$15.6 \pm 0.4$
$c$ (%)	11.93	11.94	11.95	$12.03 \pm 0.21$
$b$ (%)	15.34	15.37	15.53	$15.21 \pm 0.05$

## 5 Production of Higgs bosons at colliders

The mass of the Higgs boson is in the range 70 GeV - 140 GeV, depending on the warp factor  $z_L$ . Higgs bosons can be copiously produced at colliders at high energies. At  $\theta_H = \frac{1}{2}\pi$ , however, there emerges the  $H$  parity conservation so that Higgs bosons can be produced only in pairs, provided no other KK modes of  $P_H$  odd fields are produced. Furthermore, the Higgs boson becomes stable at  $\theta_H = \frac{1}{2}\pi$  so that conventional ways of detecting the Higgs boson, namely of finding decay products of the Higgs boson, turns out fruitless. In the current scheme the produced Higgs boson appears as missing energy and momentum. At colliders there appear at least two particles of missing energy and momentum, which makes detection hard. There is large background containing neutrinos.

An interesting feature of the present model is that the stable Higgs boson is much lighter than the KK particles, that is,  $m_H \ll m_{KK}$  as seen in Table 5. Hence it is natural to investigate the Higgs production with the effective Lagrangian among low energy fields ( $W$ ,  $Z$ , quarks and leptons) at  $\theta_H = \frac{1}{2}\pi$  [39, 19],

$$\mathcal{L}_{\text{eff}} \sim -\left\{m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2}m_Z^2 Z_\mu Z^\mu\right\} \cos^2 \frac{H}{f_H} - \sum_a m_a \bar{\psi}_a \psi_a \cos \frac{H}{f_H}. \quad (5.1)$$

Here  $f_H \sim 246$  GeV. The form of (5.1) is valid only when the relevant energy scale is sufficiently smaller than  $m_{KK}$ . For the pair production of Higgs bosons (5.1) leads to

$$\mathcal{L}_{\text{eff}} \sim \sum_a \frac{m_a}{2f_H^2} H^2 \bar{\psi}_a \psi_a + \frac{m_W^2}{f_H^2} H^2 W_\mu^\dagger W^\mu + \frac{m_Z^2}{2f_H^2} H^2 Z_\mu Z^\mu. \quad (5.2)$$

The sign of the  $H^2 W^\dagger W$  and  $H^2 Z Z$  couplings is opposite to that in SM.[36] Collider signatures of Higgs bosons in the current model have been previously investigated with this effective Lagrangian by Cheung and Song[21] and by Alves.[22]

## 5.1 Pair production of Higgs bosons at LHC

Pair production processes of Higgs bosons at LHC have been studied in Refs. [21] and [22]. Cheung and Song evaluated the cross section of the Higgs pair production associated with a  $W$  or  $Z$  boson and found that the  $ZHH(WHH)$  cross section is 0.2(0.4) fb for the case that  $m_H = 70$  GeV and the missing transverse momentum  $\cancel{p}_T$  is larger than 100 GeV. On the other hand the cross section of the background process  $ZZ \rightarrow Z\nu\bar{\nu}$  ( $WZ \rightarrow W\nu\bar{\nu}$ ) was estimated as 370(390) fb. Thus positive identification of either of the signals is virtually impossible assuming an integrated luminosity of 100 fb<sup>-1</sup>.

Alves studied the Higgs pair production in the weak boson fusion (WBF), in which the signal is a pair of forward and backward jets and a missing transverse momentum. This signal is quite similar to that of the single production of the Higgs boson decaying invisibly [53]. In Ref. [22], the signal cross section at 14 TeV LHC is estimated as 4.05(4.03) fb for  $m_H = 70(90)$  GeV using the same set of cuts employed in Ref. [53]. The background cross section is the same as that in Ref. [53] and amounts to 167 fb. Alves concluded that 255(257) fb<sup>-1</sup> is required for a  $5\sigma$  discovery.

Here we present a brief estimate of the cross section of the Higgs pair production by the WBF in the present model by relating it to the single Higgs production by the WBF in the SM. Inspecting the relevant Feynman rules in the present model and the SM, we find that  $f_H^2 |\mathcal{M}(HH)|^2 = |\mathcal{M}(h)|_{m_h^2=m_{HH}^2}^2$ , where  $h$  represents the Higgs boson in the SM,  $\mathcal{M}(HH)$  ( $\mathcal{M}(h)$ ) denotes the amplitude of the double (single) Higgs production by the WBF process in the present model (SM),  $m_h$  is the Higgs boson mass in the SM,  $m_{HH}$  is the invariant mass of the pair of Higgs bosons in the present model. Taking the two-body phase space of the Higgs pair and the statistical factor due to the existence of identical particles into account, we obtain the following relation,

$$\frac{d\sigma(HH)}{dm_{HH}^2} = \frac{1}{32\pi^2 f_H^2} \sqrt{1 - \frac{4m_H^2}{m_{HH}^2}} \sigma(h) \Big|_{m_h^2=m_{HH}^2}, \quad (5.3)$$

where  $\sigma(HH)$  ( $\sigma(h)$ ) represents the cross section of the double (single) Higgs production by the WBF in the present model (SM).

The total cross section is evaluated by integrating Eq. (5.3) over  $m_{HH}^2$  in the kinematically allowed interval. The upper value of the integration region could be as large as the center of mass energy squared of  $pp$  collisions in principle. However, as mentioned above, the effective Lagrangian in Eq. (5.2) is applicable in a limited energy scale. We

choose  $4m_{KK}^2$  as the upper value for an illustration. We note that  $4m_{KK}^2 \simeq (1.7 \text{ TeV})^2$  for the case that  $z_L = 10^5$  is approximately the same as the unitarity bound of 1.8 TeV obtained in Ref. [22]. We evaluate the right-hand side of Eq. (5.3) at the parton level with CTEQ6L parton distribution function [54] and the cuts used in Refs. [22, 53]. Our numerical calculation is done by MadGraph/MadEvent [55] without hadronization or detector simulation.

The signal cross section at 14 TeV LHC is approximately 1.3 fb for  $z_L = 10^5 - 10^{15}$ . Our result is smaller by about a factor of 3 than that of Ref. [22]. Thus, an integrated luminosity of a few  $\text{ab}^{-1}$  seems to be required to observe the signal.

## 5.2 Pair production of Higgs bosons at ILC

Cheung and Song have studied the Higgs pair production process  $e^-e^+ \rightarrow ZHH$  at ILC along with the background process  $e^-e^+ \rightarrow Z\nu\bar{\nu}$ . The Feynman diagram of the signal process is depicted in Fig. 1. The integrated luminosity for a  $5\sigma$  discovery seems to be larger than several  $\text{ab}^{-1}$  at 500 GeV ILC according to their result.

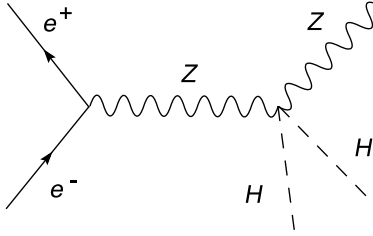


Figure 1: Pair production of Higgs bosons at ILC.

The differential cross section of  $e_R^-e_L^+ \rightarrow Z_T HH$ , where  $Z_T$  denotes the transversely-polarized  $Z$  boson, is given by

$$\frac{d\sigma_{RL}^T}{dx d\cos\theta} = \frac{g_R^2 m_Z^4 s}{2(4\pi)^3 f_H^4 (s - m_Z^2)^2} \sqrt{\frac{(x_{\max} - x)(x^2 - x_{\min}^2)}{1 + x_{\min}^2/4 - x}} \frac{1 + \cos^2\theta}{2}. \quad (5.4)$$

Here  $g_R$  denotes the coupling constant of the right-handed electron to the  $Z$  boson, which is given by  $g_R|_{SM} = -\sqrt{g^2 + g'^2} \sin^2\theta_W$  in the standard model, and  $\theta$  is the angle between the momentum of the electron and that of the  $Z$  boson in the center-of-mass system. The energy of the  $Z$  boson normalized to the beam energy,  $x$ , and its minimal and maximal values are given by

$$x = \frac{E_Z}{\sqrt{s}/2}, \quad x_{\min} = \frac{m_Z}{\sqrt{s}/2}, \quad x_{\max} = 1 - \frac{4m_H^2}{s} + \frac{x_{\min}^2}{4}. \quad (5.5)$$

For  $e_R^- e_L^+ \rightarrow Z_L H H$ , where  $Z_L$  denotes the longitudinal  $Z$  boson, the differential cross section is given by

$$\frac{d\sigma_{RL}^L}{dx d\cos\theta} = \frac{g_R^2 m_Z^4 s}{2(4\pi)^3 f_H^4 (s - m_Z^2)^2} \sqrt{\frac{(x_{\max} - x)(x^2 - x_{\min}^2)}{1 + x_{\min}^2/4 - x}} \frac{x^2}{x_{\min}^2} \frac{1 - \cos^2\theta}{2}. \quad (5.6)$$

For the case of  $e_L^- e_R^+$ ,  $g_R$  should be replaced by  $g_L$  in the above formulas. In the standard model  $g_L|_{SM} = \sqrt{g^2 + g'^2} (1/2 - \sin^2\theta_W)$ .

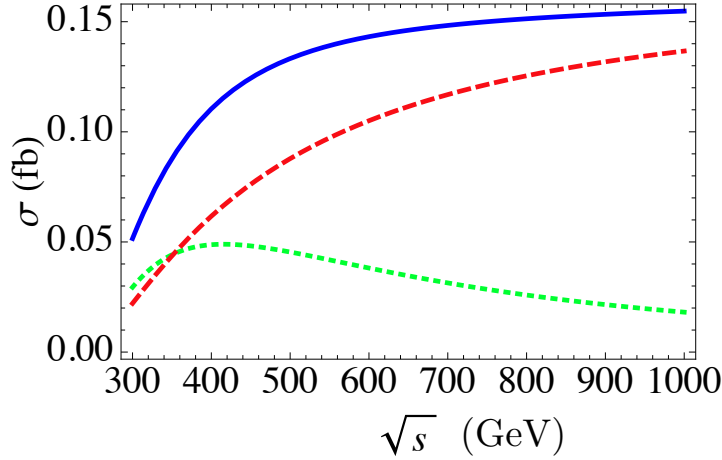


Figure 2: Cross sections of Higgs pair production at ILC for  $z_L = 10^5$ . The dotted (green) line shows the  $Z_T$  mode, the dashed (red) line is the  $Z_L$  mode and the solid (blue) represents their sum.

Figure 2 shows the total cross sections of  $e^- e^+ \rightarrow Z_T H H$  and  $e^- e^+ \rightarrow Z_L H H$  and their sum as functions of  $\sqrt{s}$  for  $z_L = 10^5$ . As  $\sqrt{s}$  increases, the cross section of the  $Z_L$  mode asymptotically becomes constant, violating the unitarity bound. This is expected because the low energy effective Lagrangian in Eq. (5.1) does not contain vertices with an odd number of Higgs fields after integrating out all heavy KK modes. With adding diagrams with such vertices to the one in Fig. 1, the leading terms in the amplitudes cancel among each other in the standard model so that the unitarity is maintained. In the present model the KK modes appearing as internal lines of the relevant diagrams are supposed to rescue the unitarity. Put it differently, the effective Lagrangian is applicable only for the case that contributions of the KK modes are negligible, that is, when  $\sqrt{s} \ll m_{KK}$ . Accordingly, unless otherwise stated, we take  $\sqrt{s} = 500$  GeV in the following numerical calculation in this subsection.

The major background is  $e^-e^+ \rightarrow Z\nu_\alpha\bar{\nu}_\alpha$  ( $\alpha = e, \mu, \tau$ ). Their total cross section is about 300 fb for  $M_{\text{mis}} \geq 120$  GeV, where  $M_{\text{mis}}$  is the invariant mass of the neutrino pair. The background is dominated by the electron neutrino mode due to the t-channel  $W$  boson exchange. In order to reduce this large background, one may use beam polarizations. We consider the limiting case of the purely right-handed electron and the purely left-handed positron as an ideal case. As well as the beam polarizations, the missing mass cut reduces the background. Corresponding to  $m_H = 72, 108, 135$  GeV for  $z_L = 10^5, 10^{10}, 10^{15}$  respectively, we take  $M_{\text{mis}} > 120, 200, 250$  GeV. We employ the GRACE system version 2 [56] in our numerical estimation of the background.

The statistical signification of the signal is defined by

$$S = \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + N_{\text{BG}}}}, \quad (5.7)$$

where  $N_{\text{signal}}$  and  $N_{\text{BG}}$  are the expected numbers of signal and background events, respectively. They are given as  $N_{\text{signal(BG)}} = L \sigma_{\text{signal(BG)}} B_{\text{ev}}$ , where  $L$  is the integrated luminosity and  $B_{\text{ev}} = 1 - B_{\text{invisible}} - B_{\tau\bar{\tau}} = 1 - 0.200 - 0.037 = 0.763$  is the effective visible branching ratio of the  $Z$  boson. The significance of the  $Z_L$  mode turns out to be much larger than that of the  $Z_T$  mode and we concentrate on the former in order to evaluate the lower bound of integrated luminosity to establish the signal. Employing  $|\cos\theta| < 0.6$ , which approximately maximizes the significance, we obtain the significance of  $e^-_R e^+_L \rightarrow Z_L H H$  as  $S/\sqrt{L} = 0.14, 0.073, 0.034$  for  $z_L = 10^5, 10^{10}, 10^{15}$  respectively, where  $L$  is in the  $\text{fb}^{-1}$  unit. Thus, in order for  $5\sigma$ , we need at least 1.3, 4.7, 21  $\text{ab}^{-1}$  for  $z_L = 10^5, 10^{10}, 10^{15}$  respectively. Since the KK mass scale rather high as  $m_{KK} = 1466$  GeV in the case of  $z_L = 10^{15}$ , one can apply the effective Lagrangian to a higher energy. For instance, we obtain  $S/\sqrt{L} = 0.11$  for  $\sqrt{s} = 750$  GeV, and the required luminosity is  $L > 2.0 \text{ ab}^{-1}$ .

## 6 Spectrum of KK states

One of the direct ways to see the extra dimension is to produce KK excited modes of various particles and observe their decays. In the current model the  $H$  parity is conserved so that  $P_H$ -odd KK modes can be produced in a pair. Production of a single KK mode occurs only for  $P_H$ -even modes. In this section we determine spectra of various KK modes.

## 6.1 KK gauge bosons

The spectrum of KK gluons  $G^{(n)}$  is identical to the spectrum of KK photons  $A^{\gamma(n)}$ . They are determined by the fourth equation in Eq. (3.6). The masses of KK  $W$  and  $Z$  bosons,  $W^{(n)}$  and  $Z^{(n)}$ , are determined by the first and second equations in Eq. (3.6), whereas those of  $W'^{(n)}$ ,  $Z'^{(n)}$  and  $A^{\hat{4}(n)}$  are determined by the third and fifth equations. The numerical values of the masses of the first five KK modes are given in Table 8.

We observe that among  $P_H$ -even modes

$$\begin{aligned}
& m_{Z^{(1)}} < m_{W^{(1)}} < m_{G^{(1)}} < m_{W^{(2)}} < m_{Z^{(2)}} \\
& < m_{Z^{(3)}} < m_{W^{(3)}} < m_{G^{(2)}} < m_{W^{(4)}} < m_{Z^{(4)}} \\
& < m_{Z^{(5)}} < m_{W^{(5)}} < m_{G^{(3)}} < m_{W^{(6)}} < m_{Z^{(6)}} < \dots
\end{aligned} \tag{6.1}$$

irrespective of  $z_L$ . Lighter the  $n = 0$  mode is, the  $n = 1$  mode becomes heavier. Masses of  $P_H$ -odd gauge bosons obey the pattern  $m_{W'^{(n)}} = m_{Z'^{(n)}} \sim m_{A^{\gamma(n)}}$  and  $m_{A^{\hat{4}(n)}} \sim m_{W^{(2n)}}$ .

Table 8: Mass spectra of KK gauge bosons in unit of GeV.

$A^{\gamma(n)}, G^{(n)}$						$W'^{(n)}, Z'^{(n)}$					
$z_L \setminus n$	1	2	3	4	5	$z_L \setminus n$	1	2	3	4	5
$10^{15}$	1144	2598	4061	5522	6991	$10^{15}$	1122	2576	4039	5503	6968
$10^{10}$	940	2125	3316	4508	5701	$10^{10}$	914	2097	3287	4479	5672
$10^5$	678	1511	2347	3184	4021	$10^5$	640	1469	2303	3139	3974

$W^{(n)}$						$A^{\hat{4}(n)}$					
$z_L \setminus n$	1	2	3	4	5	$z_L \setminus n$	1	2	3	4	5
$10^{15}$	1133	1800	2587	3285	4050	$10^{15}$	1788	3274	4748	6218	7687
$10^{10}$	927	1470	2111	2679	3301	$10^{10}$	1456	2665	5061	6257	7451
$10^5$	659	1041	1490	1889	2325	$10^5$	1020	1867	2708	3547	4384

$Z^{(n)}$					
$z_L \setminus n$	1	2	3	4	5
$10^{15}$	1130	1803	2584	3289	4046
$10^{10}$	923	1474	2107	2683	3297
$10^5$	653	1047	1484	1895	2319

## 6.2 KK quarks and leptons

The mass eigenvalue equations for quarks (3.16) and (3.17) and similar equations for leptons contain the bulk mass parameters  $c_q, c_\ell$  and the ratios  $\tilde{\mu}^q/\mu_2^q, \tilde{\mu}^\ell/\mu_3^\ell$ , which are determined

such that their running masses are given in Table 1. For the light quarks and leptons with  $c > \frac{1}{2}$  the bulk mass parameters shift to larger values for smaller  $z_L$ , whereas for the heavy quarks with  $c < \frac{1}{2}$  they become smaller.  $c_q$ ,  $c_\ell$ ,  $\tilde{\mu}^q/\mu_2^q$  and  $\tilde{\mu}^\ell/\mu_3^\ell$  are tabulated in Table 9. We note that  $\tilde{\mu}^\ell/\mu_3^\ell \ll 1$  as neutrino masses are very small.

Table 9:  $c$  and  $\tilde{\mu}/\mu_2$  for quarks and leptons.

$z_L$	$c_q$			$c_\ell$		
	$(u, d)$	$(c, s)$	$(t, b)$	$(\nu_e, e)$	$(\nu_\mu, \mu)$	$(\nu_\tau, \tau)$
$10^{15}$	0.843	0.679	0.432	0.900	0.736	0.646
$10^{10}$	1.018	0.770	0.395	1.104	0.856	0.720
$10^5$	1.548	1.049	0.268	1.721	1.222	0.948

$z_L$	$\tilde{\mu}^q/\mu_2^q$			$\mu_3^\ell/\tilde{\mu}^\ell$		
	$(u, d)$	$(c, s)$	$(t, b)$	$(\nu_e, e)$	$(\nu_\mu, \mu)$	$(\nu_\tau, \tau)$
$10^{15}$	2.283	0.0889	0.0173	$4.87 \times 10^8$	$1.14 \times 10^{10}$	$3.47 \times 10^{10}$
$10^{10}$	2.283	0.0889	0.0175	$4.87 \times 10^8$	$1.14 \times 10^{10}$	$3.47 \times 10^{10}$
$10^5$	2.283	0.0889	0.0181	$4.87 \times 10^8$	$1.14 \times 10^{10}$	$3.47 \times 10^{10}$

The spectra of KK modes of quarks and leptons are determined by the same mass eigenvalue equations as the zero modes. They are tabulated in Table 10. Except for the KK tower of  $(t, b)$ ,  $u^{(n)}$  and  $d^{(n)}$ , for instance, have approximately degenerate masses. Similarly to the case of the gauge bosons, we find an inequality  $m_{t(1)} < m_{b(1)} < m_{c(1)} \simeq m_{s(1)} < m_{d(1)} \simeq m_{u(1)}$ .

## 7 Couplings of KK gauge bosons

The couplings of quarks and leptons to KK gauge bosons can be calculated in the same manner as given in Sec. 4 for the couplings to the 4D gauge bosons. As a general feature left-handed quarks and leptons are localized near the Planck brane, whereas right-handed ones near the TeV brane. KK gauge bosons are localized near the TeV brane so that right-handed quarks and leptons have larger couplings than left-handed ones. Because of this asymmetry the left-right symmetry is broken even in the strong interaction sector.

KK gluons do not decay into massless gluons. On the other hand, KK  $W$  and  $Z$  can decay into  $WZ$  and  $WW$ , respectively.

### (i) KK photons and gluons

Table 10: The masses of KK quarks and leptons in unit of GeV.

$z_L$	$10^{15}$	$10^{10}$	$10^5$	$z_L$	$10^{15}$	$10^{10}$	$10^5$
$u^{(1)}, d^{(1)}$	1361	1203	1037	$c^{(1)}, s^{(1)}$	1249	1068	855
$u^{(2)}, d^{(2)}$	2001	1716	1383	$c^{(2)}, s^{(2)}$	1900	1593	1213
$u^{(3)}, d^{(3)}$	2823	2397	1886	$c^{(3)}, s^{(3)}$	2706	2255	1692
$u^{(4)}, d^{(4)}$	3503	2944	2258	$c^{(4)}, s^{(4)}$	3394	2812	2075
$u^{(5)}, d^{(5)}$	4287	3590	2727	$c^{(5)}, s^{(5)}$	4169	3447	2529

$z_L$	$10^{15}$	$10^{10}$	$10^5$	$z_L$	$10^{15}$	$10^{10}$	$10^5$
$t^{(1)}$	1121	912	634	$b^{(1)}$	1172	975	734
$t^{(2)}$	1797	1467	1037	$b^{(2)}$	1745	1402	936
$t^{(3)}$	2576	2097	1467	$b^{(3)}$	2627	2160	1567
$t^{(4)}$	3279	2672	1877	$b^{(4)}$	3228	2608	1778
$t^{(5)}$	4039	3287	2303	$b^{(5)}$	4090	3351	2402

$z_L$	$10^{15}$	$10^{10}$	$10^5$
$\nu_e^{(1)}, e^{(1)}, \nu_\mu^{(1)}, \mu^{(1)}, \nu_\tau^{(1)}, \tau^{(1)}$	1400	1249	1099
$\nu_e^{(2)}, e^{(2)}, \nu_\mu^{(2)}, \mu^{(2)}, \nu_\tau^{(2)}, \tau^{(2)}$	2036	1758	1441
$\nu_e^{(3)}, e^{(3)}, \nu_\mu^{(3)}, \mu^{(3)}, \nu_\tau^{(3)}, \tau^{(3)}$	2863	2445	1952
$\nu_e^{(4)}, e^{(4)}, \nu_\mu^{(4)}, \mu^{(4)}, \nu_\tau^{(4)}, \tau^{(4)}$	3540	2990	2321
$\nu_e^{(5)}, e^{(5)}, \nu_\mu^{(5)}, \mu^{(5)}, \nu_\tau^{(5)}, \tau^{(5)}$	4328	3640	2794

The couplings of the first KK photon and gluon with quarks or leptons are tabulated in Table 11. The wave functions of the KK photon and gluon are the same and their couplings to quarks are the same up to a normalization factor. The largest coupling is  $g_{u_R}^{G^{(1)}} \simeq g_{d_R}^{G^{(1)}}$ . This is different from other scenario in which the  $t$  quark dominantly couples to KK gluons.

We note that the couplings of right-handed fermions are so large that the perturbative treatment is not valid for the KK gluons. With this reservation in mind one can evaluate the decay widths of the first KK gluon by using the couplings in Table 11. The decay width is given by

$$\Gamma(G^{(n)} \rightarrow f\bar{f}) = C \frac{\alpha_s m_{G^{(n)}}}{6} F(\bar{g}_{fL}^{G^{(n)}}, \bar{g}_{fR}^{G^{(n)}}, m_f, m_{G^{(n)}}), \quad (7.1)$$

where  $F$  is defined in (4.7) and  $\bar{g}_{fL}^{G^{(n)}} = g_{fL}^{G^{(n)}}/(g_C/\sqrt{L})$ . The color factor  $C = 3$ . Numerical values are tabulated in Table 12. It is found that the decay rate to the light quarks is large. The total decay width of  $G^{(1)}$  turns out much larger than its mass. Thus the KK gluon cannot be identified as a resonance.

Table 11: The couplings of the first KK photon to leptons and quarks,  $g_f^{\gamma^{(1)}}/(g_A/\sqrt{L})$ .  $e = (g_A/\sqrt{L})\sin\theta_W$ . The couplings of the first KK gluon to quarks,  $g_f^{G^{(1)}}/(g_C/\sqrt{L})$ , are the same as  $g_f^{\gamma^{(1)}}/(g_A/\sqrt{L})$ .

$z_L$	$e_L$	$\mu_L$	$\tau_L$	$e_R$	$\mu_R$	$\tau_R$
$10^{15}$	-0.195	-0.195	-0.195	6.408	6.147	5.981
$10^{10}$	-0.241	-0.241	-0.241	5.426	5.153	4.968
$10^5$	-0.347	-0.347	-0.346	4.123	3.872	3.672

$z_L$	$u_L$	$c_L$	$t_L$	$u_R$	$c_R$	$t_R$
$10^{15}$	-0.195	-0.195	0.442	6.323	6.044	5.603
$10^{10}$	-0.241	-0.241	0.554	5.339	5.040	4.497
$10^5$	-0.347	-0.347	0.890	4.049	3.753	2.925

$z_L$	$d_L$	$s_L$	$b_L$	$d_R$	$s_R$	$b_R$
$10^{15}$	-0.195	-0.195	0.661	6.323	6.044	5.500
$10^{10}$	-0.241	-0.241	0.797	5.339	5.040	4.376
$10^5$	-0.347	-0.347	1.111	4.049	3.753	2.786

The decay width of the first KK photon  $A^{\gamma^{(1)}}$  is evaluated similarly. The decay width to a fermion pair is

$$\Gamma(\gamma^{(n)} \rightarrow f\bar{f}) = C \frac{q_f^2 \alpha m_{\gamma^{(n)}}}{3 \sin^2 \theta_W} F(\bar{g}_{fL}^{\gamma^{(n)}}, \bar{g}_{fR}^{\gamma^{(n)}}, m_f, m_{\gamma^{(n)}}), \quad (7.2)$$

where  $\bar{g}_{fL}^{\gamma^{(n)}} = g_{fL}^{\gamma^{(n)}}/(g_A/\sqrt{L})$  and  $q_f$  is a charge  $\frac{2}{3}, -\frac{1}{3}, -1, 0$ .  $C = 1$  for leptons.

In addition to decay into  $q\bar{q}$  and  $\ell\bar{\ell}$ , the first KK photon can decay into  $W^+W^-$  through

$$\begin{aligned} \mathcal{L}_{\text{int}}^{WW\gamma^{(n)}} &= ig_{WW\gamma^{(n)}} \left\{ (\partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger) W^\mu A^{\gamma^{(n)\nu}} \right. \\ &\quad \left. - (\partial_\mu W_\nu - \partial_\nu W_\mu) W^{\dagger\mu} A^{\gamma^{(n)\nu}} + (\partial_\mu A_\nu^{\gamma^{(n)}} - \partial_\nu A_\mu^{\gamma^{(n)}}) W^{\dagger\mu} W^\nu \right\}, \\ g_{WW\gamma^{(n)}} &= g_A \int \frac{dz}{kz} \left[ h_{\gamma^{(n)}}^L \left\{ (h_W^L)^2 + \frac{1}{2} (\hat{h}_W^L)^2 \right\} + h_{\gamma^{(n)}}^R \left\{ (h_W^R)^2 + \frac{1}{2} (\hat{h}_W^R)^2 \right\} \right]. \end{aligned} \quad (7.3)$$

Inserting the mode functions in Appendix A, one finds

$$g_{WW\gamma^{(n)}} = \frac{e\sqrt{L}}{\sqrt{r_{\gamma^{(n)}}} r_W} \int \frac{dz}{kz} C(z; \lambda_{\gamma^{(n)}}) \left\{ C(z; \lambda_W)^2 + \hat{S}(z; \lambda_W)^2 \right\}. \quad (7.4)$$

Note that the photon coupling is universal;  $g_{WW\gamma} = g_{WW\gamma^{(0)}} = e$ . The first KK photon has a coupling  $g_{WW\gamma^{(1)}}/e = (-0.05603, -0.06765, -0.09145)$  for  $z_L = (10^{15}, 10^{10}, 10^5)$ . The decay width is given by [50]

$$\Gamma(\gamma^{(n)} \rightarrow W^+W^-) = \frac{g_{WW\gamma^{(n)}}^2 m_{\gamma^{(n)}}}{192\pi\eta_n^2} (1 + 20\eta_n + 12\eta_n^2)(1 - 4\eta_n)^{3/2} \quad (7.5)$$

Table 12: First KK gluon decay: the branching fraction and the total width at the tree level without QCD corrections.

$z_L$	$10^{15}$	$10^{10}$	$10^5$
$u$ (%)	18.68	19.40	20.88
$d$ (%)	18.68	19.40	20.88
$s$ (%)	17.07	17.29	17.96
$c$ (%)	17.07	17.29	17.96
$b$ (%)	14.33	13.44	11.38
$t$ (%)	14.17	13.19	10.93
$\Gamma_{\text{total}}$ (GeV)	7205	4070	1576
mass (GeV)	1144	940	678

where  $\eta_n = m_W^2/m_{\gamma^{(n)}}^2$ .

The decay widths of the first KK photon are summarized in Table 13. The observed mass  $m_W$  is used in the phase space of the final state in the evaluation of  $\Gamma[\gamma^{(1)} \rightarrow W^+W^-]$ . The first KK photon  $A^{\gamma(1)}$  has a total decay width larger than or comparable to its mass. It does not look like a resonance.

## (ii) KK $W$ and $Z$

The coupling of quarks and leptons to the first KK  $W$  boson are given in Table 14. The quarks in the third generation have larger couplings than the other quarks and leptons. Couplings of right-handed quarks and leptons are rather small.

The fermion couplings of the first KK  $Z$  boson can be calculated similarly. They are tabulated in Table 15. The values of the couplings of left-handed leptons are not very sensitive to the generation. For a smaller warp factor, the magnitude of the couplings of left-handed (right-handed) leptons and quarks become larger (smaller). For left-handed leptons and quarks, the third generation has larger couplings than the first and second generations. In contrast, for right-handed leptons and quarks, the third generation has smaller couplings.

Just like KK photons KK  $Z$  bosons can decay into a pair of  $W$  bosons. Their couplings are given by

$$\begin{aligned} \mathcal{L}_{\text{int}}^{WWZ^{(n)}} = ig_{WWZ^{(n)}} \Big\{ & (\partial_\mu W_\nu^\dagger - \partial_\nu W_\mu^\dagger) W^\mu Z^{(n)\nu} \\ & - (\partial_\mu W_\nu - \partial_\nu W_\mu) W^{\dagger\mu} Z^{(n)\nu} + (\partial_\mu Z_\nu^{(n)} - \partial_\nu Z_\mu^{(n)}) W^{\dagger\mu} W^\nu \Big\} , \end{aligned}$$

Table 13: Branching fractions and decay widths of the first KK photon  $\gamma^{(1)}$ .  $\alpha = 1/128$  is used.

$z_L$	$10^{15}$	$10^{10}$	$10^5$
$e$ (%)	13.5	14.0	14.8
$\mu$ (%)	12.5	12.6	13.1
$\tau$ (%)	11.8	11.7	11.8
$u$ (%)	18.2	18.8	19.8
$c$ (%)	16.7	16.7	17.0
$t$ (%)	13.8	12.8	10.4
$d$ (%)	4.56	4.69	4.95
$s$ (%)	4.16	4.18	4.26
$b$ (%)	3.49	3.25	2.69
$W$ (%)	1.30	1.28	1.23
$\Gamma[\text{all } f\bar{f}]$ (GeV)	1933	1105	441
$\Gamma[W^+W^-]$ (GeV)	25.5	14.3	5.5
$\Gamma_{\text{total}}$ (GeV)	1959	1120	446
mass (GeV)	1144	940	678

$$g_{WWZ^{(n)}} = g_A \int \frac{dz}{kz} \left[ h_{Z^{(n)}}^L \left\{ (h_W^L)^2 + \frac{1}{2} (h_W^\wedge)^2 \right\} + h_{Z^{(n)}}^R \left\{ (h_W^R)^2 + \frac{1}{2} (h_W^\wedge)^2 \right\} \right. \\ \left. + h_{Z^{(n)}}^\wedge (h_W^L + h_W^R) h_W^\wedge \right], \quad (7.6)$$

where indices  $\mu, \nu$  are contracted with  $\eta_{\mu\nu}$ . With mode functions inserted,

$$\frac{g_{WWZ^{(n)}}}{\frac{g_A}{\sqrt{L}} \cos \theta_W} \equiv I_{WWZ^{(n)}} \\ = \frac{\sqrt{L}}{\sqrt{2r_{Z^{(n)}}} r_W} \int \frac{dz}{kz} \left[ \frac{1 - 2 \sin^2 \theta_W}{\cos^2 \theta_W} C(z; \lambda_{Z^{(n)}}) \left\{ C(z; \lambda_W)^2 + \hat{S}(z; \lambda_W)^2 \right\} \right. \\ \left. + \frac{2}{\cos^2 \theta_W} \hat{S}(z; \lambda_{Z^{(n)}}) C(z; \lambda_W) \hat{S}(z; \lambda_W) \right], \quad (7.7)$$

With the couplings  $g_{WWZ^{(n)}}$  the partial decay width  $\Gamma(Z^{(n)} \rightarrow W^+W^-)$  is given by the formula (7.5) where  $g_{WW\gamma^{(n)}}$  and  $m_{\gamma^{(n)}}$  are replaced by  $g_{WWZ^{(n)}}$  and  $m_{Z^{(n)}}$ , respectively. The enhancement factor  $1/\eta_n^2 = (m_{Z^{(n)}}/m_W)^4$  represents that  $Z^{(n)}$  decays dominantly to the longitudinal components of  $W$  over the transverse components.

The numerical values of the couplings  $g_{WWZ^{(n)}}$  are tabulated in Table 16.  $g_{WWZ} \equiv$

Table 14: The couplings of the first KK  $W$  boson with quarks and leptons,  $g_f^{W^{(1)}}\sqrt{L}/g_A$ .

$z_L$	$u_L d_L$	$c_L s_L$	$t_L b_L$
$10^{15}$	-0.138	-0.138	0.492
$10^{10}$	-0.170	-0.170	0.609
$10^5$	-0.244	-0.244	0.934

$z_L$	$e_L \nu_{eL}$	$\mu_L \nu_{\mu L}$	$\tau_L \nu_{\tau L}$
$10^{15}$	-0.138	-0.138	-0.138
$10^{10}$	-0.170	-0.170	-0.170
$10^5$	-0.244	-0.244	-0.244

$z_L$	$u_R d_R$	$c_R s_R$	$t_R b_R$
$10^{15}$	$1.02 \times 10^{-12}$	$1.08 \times 10^{-8}$	0.000308
$10^{10}$	$1.69 \times 10^{-12}$	$1.88 \times 10^{-8}$	0.000596
$10^5$	$3.66 \times 10^{-12}$	$4.55 \times 10^{-8}$	0.00204

$g_{WWZ^{(0)}}$  has been evaluated in [16]. There appears tiny deviation in  $g_{WWZ}$  from that in the SM. The couplings of KK  $Z$  are found very small;  $|g_{WWZ^{(n)}}| \ll g_{WWZ}$ .

The decay width of the first KK  $Z$  boson is tabulated in Table 17. The mass and total decay width of  $Z^{(1)}$  are 1130 GeV and 422 GeV for  $z_L = 10^{15}$ , respectively. The branching fraction of the  $WW$  mode is about 7%. (The observed mass  $m_W$  is used in the phase space of the final state in the evaluation of  $\Gamma[Z^{(1)} \rightarrow W^+W^-]$ .) In contrast to the decay width of  $Z$  boson given in Table 7, the decay rates for neutrinos in the first KK  $Z$  boson decay are very small.

## 8 Signals of KK $Z$ at Tevatron and LHC

The KK  $Z$  boson can be produced at Tevatron and LHC. We first consider the production process of the first KK  $Z$  boson ( $Z^{(1)}$ ) followed by its decay into an electron and a positron,  $q\bar{q} \rightarrow Z^{(1)} \rightarrow e^+e^-$ , as shown in Fig. 3. To this process the first KK photon ( $A^{\gamma(1)}$ ) also contributes, which has a mass close to that of  $Z^{(1)}$ . Unlike  $Z^{(1)}$ , however,  $A^{\gamma(1)}$  has a decay width larger than its mass so that its contribution is expected to give an additional smooth background to the  $Z^{(1)}$  signal. Effects of KK particles such as  $A^{\gamma(n)}$  ( $n \geq 2$ ) are ignored in our analysis for simplicity, though they also give smooth background. Our numerical calculation is done by MadGraph/MadEvent [55] at the parton level with CTEQ6L parton distribution function [54] and without detector simulation.

Table 15: The couplings of the first KK  $Z$  boson to leptons and quarks,  $g_f^{Z^{(1)}}\sqrt{L}/g_A$ .

$z_L$	$\nu_{eL}$	$\nu_{\mu L}$	$\nu_{\tau L}$	$\nu_{eR}$	$\nu_{\mu R}$	$\nu_{\tau R}$
$10^{15}$	-0.0577	-0.0577	-0.0576	$1.1 \times 10^{-31}$	$1.0 \times 10^{-29}$	$3.5 \times 10^{-28}$
$10^{10}$	-0.0712	-0.0712	-0.0711	$1.8 \times 10^{-31}$	$1.7 \times 10^{-29}$	$6.1 \times 10^{-28}$
$10^5$	-0.1025	-0.1025	-0.1023	$3.8 \times 10^{-31}$	$4.0 \times 10^{-29}$	$1.5 \times 10^{-27}$

$z_L$	$e_L$	$\mu_L$	$\tau_L$	$e_R$	$\mu_R$	$\tau_R$
$10^{15}$	0.0311	0.0310	0.0311	2.516	2.420	2.352
$10^{10}$	0.0384	0.0383	0.0384	2.126	2.033	1.953
$10^5$	0.0557	0.0553	0.0558	1.598	1.531	1.436

$z_L$	$u_L$	$c_L$	$t_L$	$u_R$	$c_R$	$t_R$
$10^{15}$	-0.0400	-0.0400	-0.2058	-1.656	-1.585	-1.467
$10^{10}$	-0.0493	-0.0493	-0.2553	-1.396	-1.320	-1.174
$10^5$	-0.0713	-0.0713	-0.3814	-1.048	-0.977	-0.743

$z_L$	$d_L$	$s_L$	$b_L$	$d_R$	$s_R$	$b_R$
$10^{15}$	0.0488	0.0488	-0.5581	0.828	0.792	0.723
$10^{10}$	0.0602	0.0602	-0.6710	0.698	0.660	0.576
$10^5$	0.0869	0.0869	-0.9219	0.524	0.488	0.371

The cross sections of  $p\bar{p} \rightarrow e^+e^-X$  at  $\sqrt{s} = 1.96$  TeV are evaluated as 22, 7.1 and 3.8 pb for  $z_L = 10^5, 10^{10}$  and  $10^{15}$ , respectively, where the invariant mass of the charged leptons is required to be larger than 150 GeV, and other cuts are the default values of MadGraph/MadEvent:  $p_T > 10$  GeV,  $|\eta| < 2.5$ , and  $\Delta R > 0.4$  for the charged leptons. In the current model the production rate of  $Z^{(1)}$  decreases for larger  $z_L$  as it becomes heavier. The background cross section, that is, the Drell-Yan cross section in the SM is 0.73 pb. Including 10% theoretical uncertainty in the signal estimation, we obtain the statistical significance at Tevatron with the integrated luminosity of  $5.4 (2.5) \text{ fb}^{-1}$ , which corresponds

Table 16: The couplings  $WWZ^{(n)}$ . The ratios  $I_{WWZ^{(n)}} = g_{WWZ^{(n)}}/(g_A \cos \theta_W/\sqrt{L})$  are listed.  $n = 0$  corresponds to the  $WWZ$  coupling.

$z_L$	$10^{15}$	$10^{10}$	$10^5$
$WWZ$	0.99985	0.99966	0.99862
$WWZ^{(1)}$	-0.0343	-0.0422	-0.0604
$WWZ^{(2)}$	$2.07 \times 10^{-5}$	$3.35 \times 10^{-5}$	$5.42 \times 10^{-5}$
$WWZ^{(3)}$	$-1.25 \times 10^{-3}$	$-1.55 \times 10^{-3}$	$-2.26 \times 10^{-3}$
$WWZ^{(4)}$	$-1.38 \times 10^{-5}$	$-2.59 \times 10^{-5}$	$-7.76 \times 10^{-5}$
$WWZ^{(5)}$	$-2.04 \times 10^{-4}$	$-2.50 \times 10^{-4}$	$-3.56 \times 10^{-4}$

Table 17: First KK  $Z$  boson decay: the branching fractions and decay widths.  $\alpha = 1/128$  is used.

$z_L$	$10^{15}$	$10^{10}$	$10^5$
$e$ (%)	12.4	12.5	11.8
$\mu$ (%)	11.5	11.4	10.9
$\tau$ (%)	10.9	10.6	9.56
$\nu_e + \nu_\mu + \nu_\tau$ (%)	0.02	0.04	0.15
$u$ (%)	16.7	16.8	15.9
$c$ (%)	15.3	15.0	13.8
$t$ (%)	12.9	11.9	9.51
$d$ (%)	4.20	4.23	4.06
$s$ (%)	3.85	3.79	3.55
$b$ (%)	5.09	6.74	14.2
$W$ (%)	7.10	6.96	6.51
$\Gamma[W^+W^-]$ (GeV)	30.0	17.0	6.8
$\Gamma_{\text{total}}$ (GeV)	422	245	104
mass (GeV)	1130	923	653

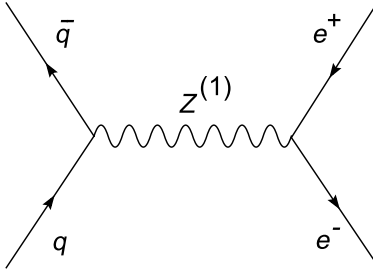


Figure 3: The first KK  $Z$  boson signal.

to the analysis by D0 collaboration [57] (CDF collaboration [58]), as 9.7 (9.7), 9.0 (8.9) and 8.1 (8.0) for  $z_L = 10^5$ ,  $10^{10}$  and  $10^{15}$ , respectively.

The first KK  $Z$  corresponds to what is referred to as  $Z'$  in the analysis of Tevatron data [57, 58]. So far no signal of  $Z'$  has been found, which gives a constraint on the present model. The signals expected at Tevatron are depicted in Fig. 4. A peak structure due to the first KK  $Z$  boson is seen in the case of  $z_L = 10^5$ , and thus the scenario with  $z_L = 10^5$  is excluded. Furthermore, although the KK  $Z$  resonance shape is smeared out by the broad contribution of the first KK photon, the other scenarios with  $z_L = 10^{10}$  and  $10^{15}$  also seem disfavored by the present Tevatron data based only on the total cross section as stated above. If we take the detailed invariant mass distribution of the lepton pair and/or the

dimuon channel into account, the limit on the warp factor will be strengthened.

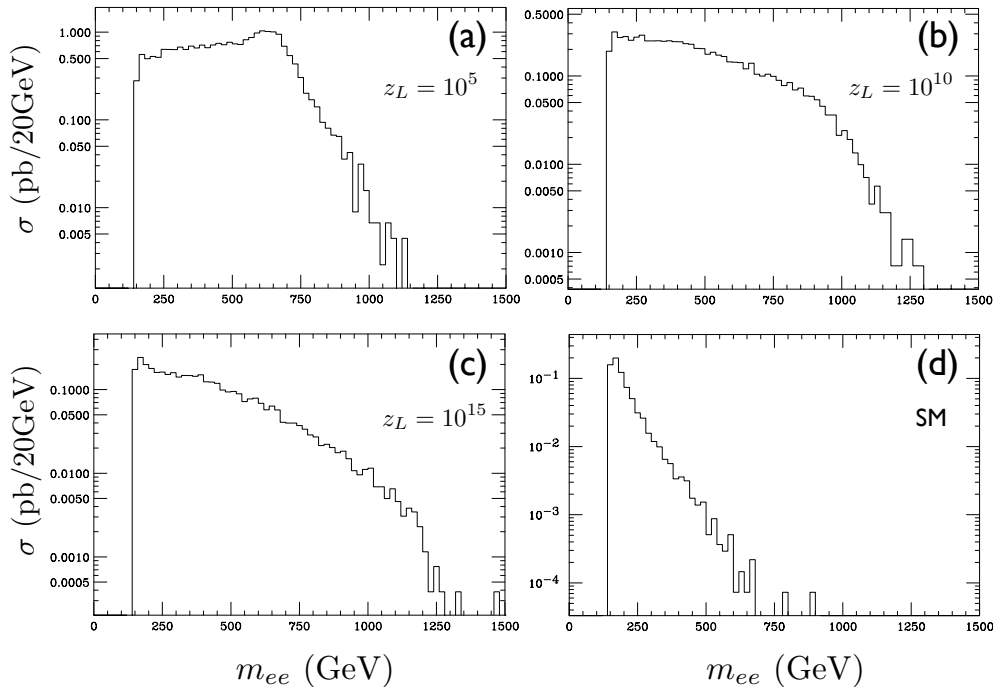


Figure 4: Distributions of the  $e^+e^-$  invariant mass in  $p\bar{p} \rightarrow e^+e^-X$  at  $\sqrt{s} = 1.96$  TeV. (a) The present model with  $z_L = 10^5$ . (b)  $z_L = 10^{10}$ . (c)  $z_L = 10^{15}$ . (d) The SM.

As for LHC, we obtain the cross sections of  $pp \rightarrow e^+e^-X$  at  $\sqrt{s} = 7$  TeV as 91, 36 and 20 pb for  $z_L = 10^5$ ,  $10^{10}$  and  $10^{15}$  respectively, and 1.8 pb for the SM. The same cuts on the final state as the  $p\bar{p}$  case are applied. The statistical significance at LHC is summarized in Table 18, where 10% theoretical uncertainty is assumed. The signals expected at LHC are shown in Fig. 5. The resonant structure of the first KK Z boson remains for all the three values of  $z_L$ .

Recently, CMS and ATLAS collaborations have searched for narrow resonances in dilepton channels and found no significant deviation from the SM [59, 60]. The integrated luminosity for the electron channel is reported as  $35 \text{ pb}^{-1}$  by CMS and  $39 \text{ pb}^{-1}$  by ATLAS. Accordingly, the cases that  $z_L \lesssim 10^{15}$  seems unlikely although we need a detailed analysis to determine the excluded parameter region. It should be noted that the total decay width of the first KK Z is very large in the current model, whereas a narrow width (3 % of its mass or less) has been assumed in the analysis in Refs. [59, 60].

We comment that contributions from higher KK photons  $A^{\gamma(n)}$  ( $n \geq 2$ ), which have broad decay widths, may have destructive interference with that from the first KK photon

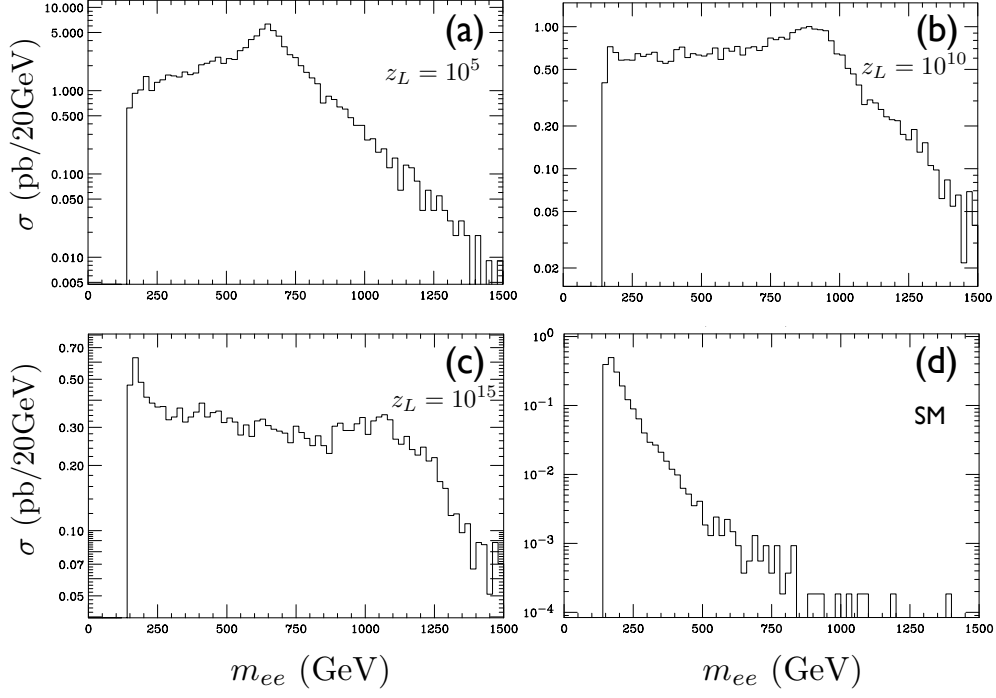


Figure 5: Distributions of the  $e^+e^-$  invariant mass in  $pp \rightarrow e^+e^-X$  at  $\sqrt{s} = 7$  TeV. (a) The present model with  $z_L = 10^5$ . (b)  $z_L = 10^{10}$ . (c)  $z_L = 10^{15}$ . (d) The SM.

so that the magnitude of the smooth background is significantly decreased. If this is the case, the bound from the current data at Tevatron and at LHC is weakened. More thorough study is necessary on this respect, which is reserved for future.

Table 18: Significance of  $pp \rightarrow e^+e^-X$  at  $\sqrt{s} = 7$  TeV.

$z_L$	$10^5$	$10^{10}$	$10^{15}$
$L = 35 \text{ pb}^{-1}$	9.7	9.1	8.5
$L = 100 \text{ pb}^{-1}$	9.7	9.4	8.9
$L = 1000 \text{ pb}^{-1}$	9.8	9.5	9.1

As seen in Tables 11 and 15, the couplings of  $A^{\gamma(1)}$  and  $Z^{(1)}$  to the right-handed fermions except the neutrinos and the bottom quark are significantly larger than those to the left-handed fermions. Such a parity violation affects the distribution of the leptons in the final state. Consider a favored parton-parton collision, for instance,  $u_R \bar{u}_R \rightarrow e_R^- e_R^+$ . The direction of the final  $e_R^-$  tends to be that of the initial  $u_R$  because of the helicity conservation. This angular distribution in the parton center-of-mass frame results in a harder electron

spectrum (and a softer positron spectrum) in the  $pp$  center-of-mass frame since most of the initial quark-antiquark pairs are boosted in the direction of the initial quark in the  $pp$  collider. Hence, we expect a wider rapidity distribution for the electron than the positron. We present, in Fig. 6, the rapidity ( $y$ ) distributions of the electron and positron in the present model with  $z_L = 10^{15}$  and in the SM. Though the both models have the similar tendency that the electron distribution is wider than the positron, the difference between the electron and the positron is more significant in the present model. This feature in the rapidity distributions is quantified by the central charge asymmetry [61],

$$A_{cc}(y_c) = \frac{\sigma(|y_{e-}| < y_c) - \sigma(|y_{e+}| < y_c)}{\sigma(|y_{e-}| < y_c) + \sigma(|y_{e+}| < y_c)}. \quad (8.1)$$

Our numerical study suggests that the statistical significance of  $A_{cc}(y_c)$  is maximized with  $y_c \sim 0.6$  for the case of  $z_L = 10^{15}$ . We find that  $A_{cc}(0.6) = -0.32(-0.17)$  for  $z_L = 10^{15}$  (the SM) and the significance of  $5\sigma$  is expected with the integrated luminosity of about  $1 \text{ fb}^{-1}$ . Another signal of the parity violation may be seen in the lepton forward-backward asymmetry with respect to the boost direction of the KK  $Z$  boson [62, 63].

We also evaluate the cross section of  $pp \rightarrow je^+e^-X$  at  $\sqrt{s} = 7 \text{ TeV}$ , where  $j$  denotes a jet, to find 43, 17 and 9.3 pb for  $z_L = 10^5$ ,  $10^{10}$  and  $10^{15}$  respectively. The same cuts on the final leptons as the  $pp \rightarrow e^+e^-X$  case are applied, and the default cuts for jets in MadGraph/MadEvent, that is,  $p_T > 20 \text{ GeV}$ ,  $|\eta| < 5$  and  $\Delta R > 0.4$  for the jet are employed. Approximately 80% of the cross section for  $z_L = 10^{15}$  includes a gluon jet ( $q\bar{q} \rightarrow gZ^{(1)}, gA^{\gamma(1)}$ ) and the rest does a quark jet ( $qg \rightarrow qZ^{(1)}, qA^{\gamma(1)}$ ). The SM cross section is estimated to be 0.66 pb. The statistical significance assuming 10% theoretical uncertainty is shown in Table 19. The  $pp \rightarrow je^+e^-X$  mode has a comparable sensitivity of the  $pp \rightarrow e^+e^-X$  mode from the statistical point of view, but the background should be studied carefully since the signal is more complicated.

Table 19: Significance of  $pp \rightarrow je^+e^-X$  at  $\sqrt{s} = 7 \text{ TeV}$ .

$z_L$	$10^5$	$10^{10}$	$10^{15}$
$L = 35 \text{ pb}^{-1}$	9.5	8.9	8.1
$L = 100 \text{ pb}^{-1}$	9.7	9.3	8.8
$L = 1000 \text{ pb}^{-1}$	9.8	9.6	9.2

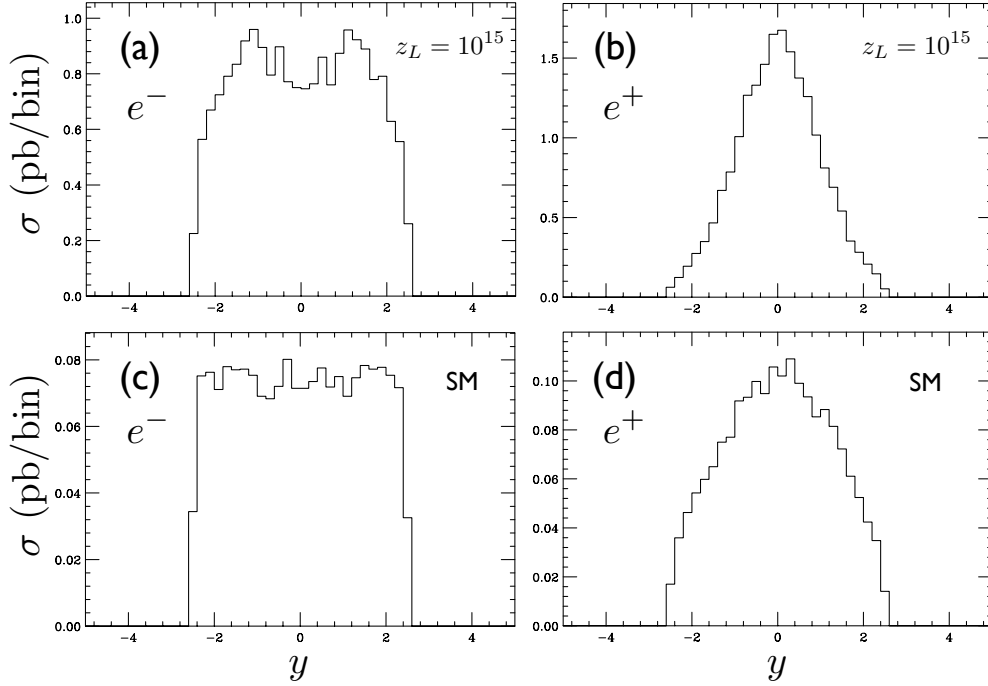


Figure 6: The rapidity distributions. (a) The electron distribution in the present model with  $z_L = 10^{15}$ . (b) The positron distribution in the present model with  $z_L = 10^{15}$ . (c) The electron distribution in the SM. (d) The positron distribution in the SM.

## 9 Conclusions

In the present paper we have explored collider signatures of the  $SO(5) \times U(1)$  gauge-Higgs unification model in the RS space. The model predicts  $\theta_H = \frac{1}{2}\pi$  and the stable Higgs boson. The gauge and Higgs couplings of quarks and leptons deviate from those in the standard model. With the warp factor  $z_L$  given, the mass spectra and couplings of all fields are determined.

There arises small deviation in the gauge couplings of quarks and leptons. They lead to forward-backward asymmetry in  $e^+e^-$  annihilation on the  $Z$  resonance. It was found that the gauge-Higgs unification model gives good fit to the forward-backward asymmetry data in a wide range of  $z_L$ . However, the data of branching fractions of various modes in the  $Z$  boson decay is fit well only for  $z_L \gtrsim 10^{15}$ .

Pair production of Higgs bosons at ILC,  $e^+e^- \rightarrow ZHH$ , is marginal. There is large background containing neutrinos. With polarized beam and appropriate cut, the statistical significance  $S$  of the signal is estimated to be  $S/\sqrt{L(\text{fb}^{-1})} = 0.11$  for  $\sqrt{s} = 750 \text{ GeV}$  for

$z_L = 10^{15}$ , which requires the luminosity  $L > 2.0 \text{ ab}^{-1}$  for  $5\sigma$  discovery.

Another important way to test the model is to produce KK modes. The production of the first KK  $Z$  boson  $Z^{(1)}$  decaying into  $e^+e^-$  gives a clear signal. At  $z_L = 10^{15}$  the mass and width of  $Z^{(1)}$  are about 1130 GeV and 422 GeV, respectively. The production of  $Z^{(1)}$  can be discovered at LHC through  $pp \rightarrow Z^{(1)}X \rightarrow e^+e^-X$  with  $100 \text{ pb}^{-1}$ . There appears a smooth background coming from the production and decay of KK photons. The mass and width of the first KK photon  $A^{\gamma(1)}$  are 1144 GeV and 1959 GeV at  $z_L = 10^{15}$ , respectively. We have evaluated the cross section including the contribution from  $A^{\gamma(1)}$ . The present limit from the  $Z'$  searches at Tevatron and LHC excludes  $z_L \lesssim 10^{15}$ . However, a more thorough study taking account of contributions of higher KK photons  $A^{\gamma(n)}$  ( $n \geq 2$ ) is necessary, as destructive interference could occur in the smooth background.

It is a general feature that KK gluons, photons and  $Z$  couple dominantly to right-handed quarks and leptons. The large parity violation affects the rapidity distributions of  $e^+$  and  $e^-$  in the decay of  $Z^{(1)}$ , which is quantified by measuring the central charge asymmetry.

We conclude that the present precision data of the gauge couplings and  $Z'$  search indicates a large warp factor  $z_L > 10^{15}$ .  $Z^{(1)}$  production at LHC is a promising way to test the model.

In the present paper we have investigated the  $SO(5) \times U(1)$  gauge-Higgs model with bulk fermions in the vector representation of  $SO(5)$ , in which right-handed quarks and leptons are localized near the TeV brane and have large couplings with KK gauge bosons. It is interesting to see whether the couplings of the leptons to the KK photons can be suppressed by introducing bulk lepton multiplets in other tensorial representation of  $SO(5)$ .

It has been shown that in order for the stable Higgs boson to account for the dark matter of the universe, its mass must be  $m_H \sim 70 \text{ GeV}$ , which is obtained with a small warp factor  $z_L \sim 10^5$  in the current model. Further improvement of the model is necessary to explain both collider data and dark matter.

## Acknowledgements

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## A Normalized mode functions

In this appendix, mode functions with their normalization factors at  $\theta_H = \frac{1}{2}\pi$  are collected.[18] Basis functions are given in (3.5) and (3.13). For convenience we define

$$\hat{S}(z; \lambda) = \frac{C(1; \lambda)}{S(1; \lambda)} S(z; \lambda) , \quad \hat{S}_L(z; \lambda, c) = \frac{C_L(1; \lambda, c)}{S_L(1; \lambda, c)} S_L(z; \lambda, c) . \quad (\text{A.1})$$

### Gauge bosons

Gauge bosons are expanded as in (3.4). Mode functions  $h(z)$  of  $P_H$ -even fields, for instance, satisfy orthogonality conditions

$$\begin{aligned} \int_1^{z_L} \frac{dz}{kz} \left\{ h_{W^{(n)}}^L h_{W^{(m)}}^L + h_{W^{(n)}}^R h_{W^{(m)}}^R + h_{W^{(n)}}^\wedge h_{W^{(m)}}^\wedge \right\} &= \delta^{nm} , \\ \int_1^{z_L} \frac{dz}{kz} \left\{ h_{Z^{(n)}}^L h_{Z^{(m)}}^L + h_{Z^{(n)}}^R h_{Z^{(m)}}^R + h_{Z^{(n)}}^\wedge h_{Z^{(m)}}^\wedge + h_{Z^{(n)}}^B h_{Z^{(m)}}^B \right\} &= \delta^{nm} , \\ \int_1^{z_L} \frac{dz}{kz} \left\{ h_{\gamma^{(n)}}^L h_{\gamma^{(m)}}^L + h_{\gamma^{(n)}}^R h_{\gamma^{(m)}}^R + h_{\gamma^{(n)}}^B h_{\gamma^{(m)}}^B \right\} &= \delta^{nm} , \\ \int_1^{z_L} \frac{dz}{kz} \left\{ h_{Z^{(n)}}^L h_{\gamma^{(m)}}^L + h_{Z^{(n)}}^R h_{\gamma^{(m)}}^R + h_{Z^{(n)}}^B h_{\gamma^{(m)}}^B \right\} &= 0 . \end{aligned} \quad (\text{A.2})$$

Similar relations hold for other mode functions.

#### (i) Photon tower ( $\hat{A}_\mu^\gamma$ )

$$\begin{aligned} h_{\gamma^{(n)}}^L &= h_{\gamma^{(n)}}^R = \frac{s_\phi}{\sqrt{1+s_\phi^2}} \frac{1}{\sqrt{r_{\gamma^{(n)}}}} C(z; \lambda_{\gamma^{(n)}}) , \\ h_{\gamma^{(n)}}^B &= \frac{c_\phi}{\sqrt{1+s_\phi^2}} \frac{1}{\sqrt{r_{\gamma^{(n)}}}} C(z; \lambda_{\gamma^{(n)}}) , \\ r_{\gamma^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} C(z; \lambda_{\gamma^{(n)}})^2 . \end{aligned} \quad (\text{A.3})$$

For a photon  $C(z; \lambda_{\gamma^{(0)}} = 0) = \text{const} = \sqrt{r_{\gamma^{(0)}}/L}$ . Note that  $s_\phi = \tan \theta_W$  and  $1/\sqrt{1+s_\phi^2} = \cos \theta_W$ .

#### (ii) W boson tower ( $\hat{W}_\mu$ )

$$h_{W^{(n)}}^L = h_{W^{(n)}}^R = \frac{1}{\sqrt{2r_{W^{(n)}}}} C(z; \lambda_{W^{(n)}}) ,$$

$$\begin{aligned}
h_{W^{(n)}}^\wedge &= -\frac{1}{\sqrt{r_{W^{(n)}}}} \hat{S}(z; \lambda_{W^{(n)}}) , \\
r_{W^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} \left\{ C(z; \lambda_{W^{(n)}})^2 + \hat{S}(z; \lambda_{W^{(n)}})^2 \right\} .
\end{aligned} \tag{A.4}$$

(iii) **Z boson tower** ( $\hat{Z}_\mu$ )

$$\begin{aligned}
h_{Z^{(n)}}^L &= h_{Z^{(n)}}^R = \frac{c_\phi^2}{\sqrt{1+s_\phi^2}} \frac{1}{\sqrt{2r_{Z^{(n)}}}} C(z; \lambda_{Z^{(n)}}) = \frac{1-2\sin^2\theta_W}{\cos\theta_W} \frac{C(z; \lambda_{Z^{(n)}})}{\sqrt{2r_{Z^{(n)}}}} , \\
h_{Z^{(n)}}^\wedge &= -\sqrt{1+s_\phi^2} \frac{1}{\sqrt{r_{Z^{(n)}}}} \hat{S}(z; \lambda_{Z^{(n)}}) = -\frac{1}{\cos\theta_W} \frac{\hat{S}(z; \lambda_{Z^{(n)}})}{\sqrt{r_{Z^{(n)}}}} , \\
h_{Z^{(n)}}^B &= -\frac{\sqrt{2}s_\phi c_\phi}{\sqrt{1+s_\phi^2}} \frac{1}{\sqrt{r_{Z^{(n)}}}} C(z; \lambda_{Z^{(n)}}) = -\frac{g_A \sin^2\theta_W}{g_B \cos\theta_W} \frac{\sqrt{2}C(z; \lambda_{Z^{(n)}})}{\sqrt{r_{Z^{(n)}}}} , \\
r_{Z^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} \left\{ c_\phi^2 C(z; \lambda_{Z^{(n)}})^2 + (1+s_\phi^2) \hat{S}(z; \lambda_{Z^{(n)}})^2 \right\} .
\end{aligned} \tag{A.5}$$

(iv)  $\hat{W}'_\mu$  tower

$$\begin{aligned}
h_{W'^{(n)}}^L &= -h_{W'^{(n)}}^R = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{r_{W'^{(n)}}}} C(z; \lambda_{W'^{(n)}}) , \\
r_{W'^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} C(z; \lambda_{W'^{(n)}})^2 .
\end{aligned} \tag{A.6}$$

(v)  $\hat{Z}'_\mu$  tower

$$\begin{aligned}
h_{Z'^{(n)}}^L &= -h_{Z'^{(n)}}^R = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{r_{Z'^{(n)}}}} C(z; \lambda_{Z'^{(n)}}) , \\
r_{Z'^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} C(z; \lambda_{Z'^{(n)}})^2 .
\end{aligned} \tag{A.7}$$

(vi)  $\hat{A}_\mu^{\hat{4}}$  tower

$$\begin{aligned}
h_{A^{(n)}} &= \frac{1}{\sqrt{r_{A^{\hat{4}}(n)}}} S(z; \lambda_{A^{\hat{4}}(n)}) , \\
r_{A^{\hat{4}}(n)} &= \int_1^{z_L} \frac{dz}{kz} S(z; \lambda_{A^{\hat{4}}(n)})^2 .
\end{aligned} \tag{A.8}$$

The mode functions for the fifth-dimensional component are given similarly.  $h_S^{LR}, h_D^{LR}, h_B \propto C'(z; \lambda)$  and  $h_H^\wedge, h_D^\wedge \propto S'(z; \lambda)$ . The normalization condition is given by  $\int_1^{z_L} (k dz/z) (h_s)^2 = 1$  where  $h_s = h_S^{LR}, h_H^\wedge, h_D^{LR}, h_D^\wedge, h_B$ .

## Fermions

For  $P_H$ -even  $\psi_{\frac{2}{3}(+)}^{(n)}$ , the equation (3.14) leads to

$$\left[ a_U^{(n)}, a_{t'}^{(n)} \right] \simeq \left[ -\frac{\sqrt{2}\tilde{\mu}}{\mu_2}, -\frac{C_L(1; \lambda_n, c_t)}{S_L(1; \lambda_n, c_t)} \right] a_{B+t}^{(n)}. \quad (\text{A.9})$$

With this ratio, the coefficient is given by

$$a_{B+t}^{(n)} = \left[ \int_1^{z_L} dz \left\{ \left[ 2 \left( \frac{\tilde{\mu}}{\mu_2} \right)^2 + 1 \right] C_L(z; \lambda_n, c_t)^2 + \hat{S}_L(z; \lambda_n, c_t)^2 \right\} \right]^{-1/2}. \quad (\text{A.10})$$

For  $P_H$ -odd  $t_{(-)}^{(n)}$ , the coefficient is

$$a_{B-t}^{(n)} = \left[ \int_1^{z_L} dz C_L(z; \lambda_{t_{(-)}^{(n)}}, c_t)^2 \right]^{-1/2}. \quad (\text{A.11})$$

For  $P_H$ -even  $\psi_{-\frac{1}{3}(+)}^{(n)}$ , the coefficients satisfy

$$\left[ a_b^{(n)}, a_{b'}^{(n)} \right] \simeq \left[ -\frac{\sqrt{2}\mu_2}{\tilde{\mu}}, -\frac{C_L(1; \lambda_n, c_t)}{S_L(1; \lambda_n, c_t)} \right] a_{D+X}^{(n)}, \quad (\text{A.12})$$

which yields

$$a_{D+X}^{(n)} = \left[ \int_1^{z_L} dz \left\{ \left[ 2 \left( \frac{\mu_2}{\tilde{\mu}} \right)^2 + 1 \right] C_L(z; \lambda_n, c_t)^2 + \hat{S}_L(z; \lambda_n, c_t)^2 \right\} \right]^{-1/2}. \quad (\text{A.13})$$

For  $P_H$ -odd  $b_{(-)}^{(n)}$ , the coefficient is given by

$$a_{D-X}^{(n)} = \left[ \int_1^{z_L} dz C_L(z; \lambda_{b_{(-)}^{(n)}}, c_t)^2 \right]^{-1/2}. \quad (\text{A.14})$$

To obtain overlap integrals for the gauge couplings, these normalization constants are taken into account.

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